SAINIK SCHOOL IMPHAL

SUMMER VACATION

2025-26

HOMEWORK/ASSIGNMENT/PROJECTS

CLASS XI

XI Chemistry Summer Vacation Home Work

Unit I: Some basic concepts of chemistry.

1. What are the terms used to express concentration of a solution.

2. Explain the following terms:

(i) Normality

(ii) Molarity

(iii) Molality

(iv) Mole fraction.

3. Chlorophyl, the green colouring matter of plant contains 2.68% of Mg metal by weights. Calculate total number of Mg atoms in 2g of chlorophyl. (Atomic mass of Mg = 24g. mol^{-1}).

4. 50g of NaCℓ dissolved in 150 mℓ of H_2O. Calculate the normality of the solution.

5. What is the concentration of sugar ($C_{12}H_{22}O_{11}$) in mol. L^{-1}if its 20g is dissolved in enough water to make the final volume up to 2L?

6. If the density of methanol is 0.793 kg. L^{-1}, what is the volume required to make 2.5 liter of its 0.25M solution?

7. Calculate molality of a solution of ethanol in water in which the mole fraction of ethanol is 0.040.

8. Calculate molarity of NaOH solution in which 4g of NaOH is dissolved in enough water to make the final volume 250mℓ.

9. What do you mean by empirical formula and molecular formula? A substance on analysis gave the following percentage composition of Na = 43.4%, C = 11.3% and O = 45.3%. Calculate its empirical formula (Atomic mass of Na =23, C = 12 and O = 16).

10. What do you mean by limiting reagent? Calculate amount of NH_3 obtaied when 50g of N_2 gas reacts with 10g of H_2 gas and identified the limiting reagetnt also.

11. Calculate percentage composition of each atoms present in 10g of Na_2CO_3.

END

SAINIK SCHOOL IMPHAL
CLASS-11
SUBJECT: COMPUTER SCIENCE (083)

Chapter 1

1. Name the software required to make a computer functional. Write down its two primary functions.
2. What is the need of RAM? How does it differ from ROM?
3. Differentiate between Proprietary software and freeware software. Name two software of each type
4. Convert the following into bytes
    a. 2MB
    b. 3.7 GB
    c. 1.2 TB
5. What is the security threats involved when we throw away electronic gadgets that are Non-functional?
6. Write down the type of memory needed to do the following.
    a. To store data permanently
    b. To execute the program
    c. To store the instructions which cannot be overwritten

Python Programming

Program

1. Input a welcome message and display it
2. Input two numbers and display the larger/smaller number.
3. Input three numbers and display the largest/smallest number.
4. Generate the following patterns

Pattern 1	Pattern 2	Pattern 3
*	12345	ABCDE
* *	1234	ABCD
* * *	123	ABC
* * * *	12	AB
* * * * *	1	A

5. Display the terms of a Fibonacci series

SAINIK SCHOOL IMPHAL
SESSION: 2025-26
ENGLISH CORE (301)
CLASS: XI
(SUMMER VACATION HOMEWORK)

1. What is Reading Comprehension? And why is it very important for every competitive examinations? (200 words)

2. Aged people should not be left behind and every effort should be made that they live with their children and grandchildren. This will inculcate a proper understanding between the old and the new generation. Write down your views in about 200 words with reference to "The Portrait of a Lady".

3. How do Gordon and his family demonstrate resilience in the face of adversity in the lesson "We're Not Afraid to Die If We Can All Be Together."? Reflect on their emotional impact of the experience. And how does their ordeal change their perspectives on life and their priorities? (200 words)

4. 'Fear or/and conscience' - has worked behind the boys' decision of returning the stolen horse as trust and honesty were the hallmarks of the tribe the two boys belonged to. Illustrate your answer with reference to the text "The Summer of the Beautiful White Horse". (200 words)

5. Past, whether good or bad, is gone. We must never worry about it. All we need is to make the best of our present. Explain it with reference to the story 'The Address'. (200 words)

IMPORTANT NOTES:

(i) The assignment should be done on A4 size sheets and compiled in a hard bound file/folder. And design an attractive cover for your file/folder indicating Name, Adm. No., Class, Session, Section, and Subject clearly.

(ii) All the answers should be neatly presented in your own handwriting.

(iii) Remember, a well-presented 'Holiday Homework' fetches you appreciations of the teachers and the classmates.

ENJOY THE VACATION WITH YOUR PARENTS; STAY SAFE, STAY HAPPY.

MR. M A HAQUE

MASTER (ENGLISH)

CLASS-11 :MATHS

NCERT CHAPTER 1: SETS

Definition of Set: Well defined collection of distinct objects(or elements) is called a set.

Sets are usually denoted (or named) by capitals letters A,B,C etc. and the elements of set are denoted by small letters a, b, c etc. Elements of set are separated from each other using comma (,) and are enclosed within a pair of second brackets { }.

Examples of set (i) collection of prime numbers between 1 and 10.

(ii) collections of vowels of English alphabets .

(iii) collections of solutions of equation $x^2 - 5x + 6 = 0$

Examples of not set (i) collections of best five footballers of world.

(ii) collections of intelligent students in a class.

(iii) collections of five real numbers between 3 and 7.

Some standard sets : N=$\{1,2,3,4, \dots \dots\}$=set of natural numbers .

Z(or I) =$\{\dots ., -3, -2, -1, 0, 1, 2, 3, , \dots \dots\}$=set of integers.

W =$\{0,1,2,3,4, \dots \dots\}$=set of whole numbers .

Q=set of rational numbers .

R=set of real numbers .

C=set of complex numbers .

R_+=set of positive real numbers .

R_0=set of non-zero real numbers.

Note: Let set A= $\{a, e, i, o, u\}$,then

(i) $a \in A$ (read as a belongs to A) means a is an elements of set A .

(ii) $a \notin A$ (read as a not belongs to A) means a is not an elements of set A .

(iii) $n(A) = 5$ (called cardinality of set A) means number of elements of set A.

Representation of sets:

1. **Roster method** (or tabular method or listing method): In this method we make the list of all elements that satisfy the characteristic of that set.
   Example : (i) A= set of all odd natural numbers less than 10
   $= \{1,3,5,7,9\} \longrightarrow n(A) = 5$
   (ii) B= set of all prime natural numbers
   $= \{2,3,5,7,11,13 \ldots..\} \longrightarrow n(B) = \infty$
   (iii) C=set of all letters used in word MATHEMATICS.
   $=\{M, A, T, H, E, I, C, S\} \longrightarrow n(C) = 8$

2. **Set builder method** (or rule method):Roster method is always not convenient to represent a set . For example if it is asked to write the set of all 2000 workers working in a factory ,then roster method will be very tedious and time consuming . So another method is develop for representation of set ,called set builder method.

In this method ,we represent the elements of set by $'x'$ and write the properties (or characteristics) satisfy by x and enclosed them in a pair of second brackets i.e $\{x/properties\ of\ x\}$

Example : (i) A= set of all real numbers lying between 3 and 5.
$=\{x/x \in R and\ 3 < x < 5\}$
(ii) B= set of all even natural numbers . $=\{2x/x \in N\}$
(iii) C= set of all odd natural numbers . $=\{2x + 1/x \in W\}$

Ex1. Write the set $\{x/x\ is\ positive\ integer\ and\ x^2 < 40\}$ in roster form.

Sol[n]: $\{1,2,3,4,5,6\}$

Ex2. Write the set $\{x/x \in Z, x^2 < 20\}$ in roster form.

Sol[n]: $\{-4, -3, -2, -1,0,1,2,3,4\}$

Ex3. Write the set $\{x/x \in N\ and\ \ x^2 - 25 = 0\}$ in roster form.

Sol[n]: $x^2 - 25 = 0 \Rightarrow x^2 = 25 \Rightarrow x = \pm 5$

The required set is $\{5\}$ as $-5 \notin N$

Ex 4. Write the set $\{1,4,9,16, \ldots \ldots.\}$ In rule method.

Sol[n]: $\{x^2\ /\ x \in N\}$

Types of sets :

1.**Null set (or empty set or void set) :** A set containing no elements is called a null set and is denoted by ∅ or { }

Examples: (i) Set of prime numbers between 7 and 11.

(ii) $\{x/x \in R \text{ and } x^2 = -1\}$

Note : If A is a null set ,then n(A)=0.

2.**Singleton set :** A set containing only one element is called a singleton set.

Examples: (i) Set of natural satellite of earth .

(ii) $Set\ of\ even\ prime\ number$

Note : If A is a singleton set ,then n(A)=1

3.**Finite set and infinite set** :A set is said to be finite ,if it contains finite number of elements ,otherwise it is called an infinite set.

Examples: (i) A=$\{a, e, i, o, u\}$ is a finite set as n(A)=5=finite.

(ii) ∅ is a finite set as n(∅)=0=finite.

(iii)N is an infinite set as n(N) =∞

4. **Subset and superset:** Set A is said to be a subset of set B if every element of A is also an element of B(i.e A is a part of B or A is contained in B) and B is called superset of A and symbolically it is denoted by $A \subseteq B$.

Thus $A \subseteq B$ if $a \in A \Rightarrow a \in B \ \forall a \in A$

Examples: (i) A=$\{1,2,3,4,5\}$, B=$\{5,2,1,4,3\}$

Here $A \subseteq B$ and $B \subseteq A$

(ii) A=$\{a, e, i, o, u\}$, B=$\{a, b, i, o, c, e\}$

Here $A \nsubseteq B$ as $u \notin B$ and $B \nsubseteq A$ as $b, c \notin A$

5. **Proper and improper subset:** Set A is said to be proper subset of B if $A \subseteq B$ but $B \nsubseteq A$ and is denoted by A⊂ B,otherwise A is improper subset of B .

Examples: (i) A=$\{1,2,3\}$,B= $\{1,2,3,4,5\}$

Here $A \subseteq B$ and $B \nsubseteq A$ and so A⊂ B

(ii) A=$\{1,2,3,4,5\}$, B=$\{5,2,1,4,3\}$

Here $A \subseteq B$ and $B \subseteq A$ and so A$\not\subset B$

Note: (i) Any set A is always a subset of itself and it is called improper subset.(as $A \subseteq A$ and $A \subseteq A$) .

(ii) N$\subset W \subset Q \subset R \subset C$

(iii) Null set ∅ is proper subset of any set A .

Proof(iii) If possible, let $\emptyset \not\subset A$

⇒ ∃ at least one element in ∅ which does not belong to A .

But this contradicts the definition of ∅ as ∅ has no elements .

∴ our assumption is wrong .

∴ $\emptyset \subset A$.

6. **Equal sets and equivalent sets:** Two sets A and B are said to be equal if $A \subseteq B$ and $B \subseteq A$ and is written as A=B.

Again two finite sets A and B are said to be equivalent if n(A) = n(B)

Example: A=$\{1,2,3\}$, B=$\{a, b, c\}$ are equivalent but A$\neq B$

7. **Power set:** The set of all the subsets of a set A is called the power set of A and is denoted by P(A) i.e P(A)=$\{X \,/\, X \subseteq A\}$.

Example: If A=$\{1,2,3\}$,find P(A).

Sol^n: Subsets of set A are

$S_1=\{1\}$,$S_2=\{2\}$,$S_3=\{3\}$,$S_4=\{1,2\}$,$S_5=\{1,3\}$,$S_6=\{2,3\}$,$S_7=\{1,2,3\} = A$,$S_8= \emptyset$

∴P(A)={ $S_1, S_2, S_3, S_4, S_5, S_6, S_7, S_8$}

$= \{\{1\},\{2\},\{3\},\{1,2\},\{1,3\},\{2,3\},\{1,2,3\},\emptyset\}$

Note: If n(A)= n ,then

(i) $n[P(A)] = 2^n$

(ii) number of proper subsets of A=2^n-1

(iii) number of non empty proper subsets of A=2^n-2

8.Universal set: A set from which all the subsets are taken in a given discussion is called universal set and is denoted by U.

Example:

(i) For the set of integers, set of prime numbers ,set of rational numbers etc. the set of real numbers can be considered as universal set.

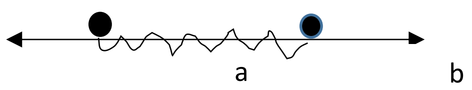
(ii) For set A= $\{a, b, c\}$,B=$\{a, e, i, o, u\}$,C=$\{p, q, x, y\}$, the set of letters in English alphabets can be considered as the universal set U.

9.**Disjoint sets:** Two or more sets are said to be disjoint if they have no common elements .

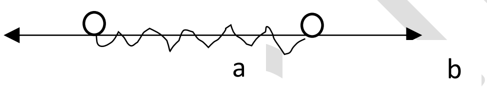
Example: A=$\{1,2,3\}$ and B=$\{4,5,6,7\}$ are disjoint sets.

10. **Intervals(subsets of real numbers):**Let $a, b \in R\ \ and\ a < b$,then the following sets are called intervals as follow.

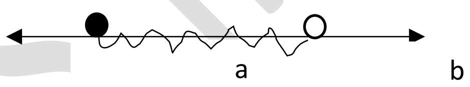
(i) $\{x\ /\ a \le x \le b\} = [a, b]$ is called closed interval.



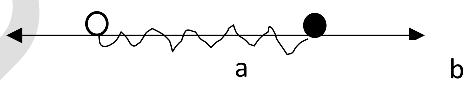
(ii) $\{x\ /\ a < x < b\} = (a, b)$ is called open interval.



(iii) $\{x\ /\ a \le x < b\} = [a, b)$ is called left closed right open interval.

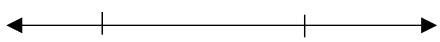


(iv) $\{x\ /\ a < x \le b\} = (a, b]$ is called left open right closed interval.



Sign of product of two or more linear factors: $f(x)\ or\ y = (x-a)(x-b)$

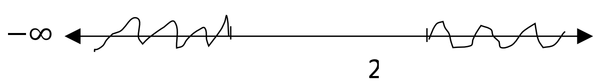
1. $f(x)\ or\ y = (x-2)(x-7) > 0$



CaseI: $(x-2) > 0$ and $(x-7) > 0$

$\Rightarrow x > 2$ and $x > 7$

The common solution is $x > 7$



CaseII: $(x-2)<0$ and $(x-7)<0$

$\Rightarrow x<2$ and $x<7$

The common solution is $x<2$

Combining Case:I and Case:II, the solutions for $(x-2)(x-7)>0$

are $(-\infty,2)\cup(7,\infty)$

2. f(x) or y = $(x-2)(x-7)<0$

CaseI: $(x-2)>0$ and $(x-7)<0$

$\Rightarrow x>2$ and $x<7$

The common solution is $2<x<7$

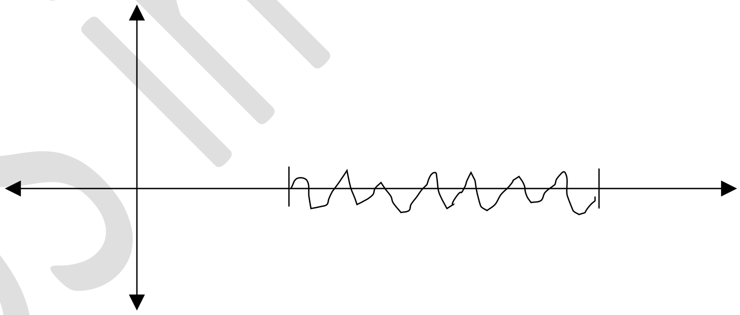
CaseII: $(x-2)<0$ and $(x-7)>0$

$\Rightarrow x<2$ and $x>7$

There is no common solution .

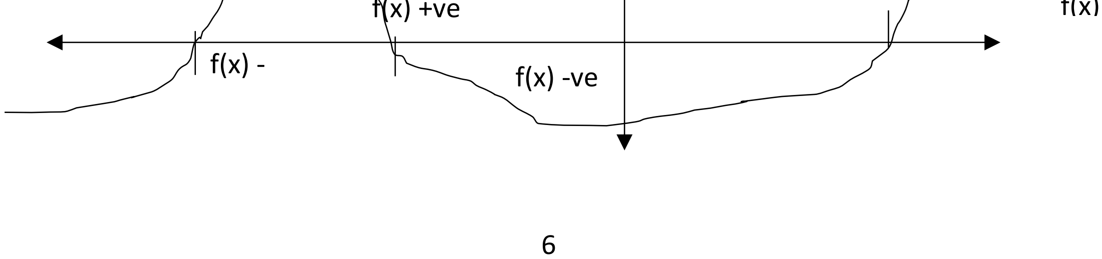
Combining CaseI and CaseII, the solutions for $(x-2)(x-7)<0$

are $(2,7)$



Example 1: $f(x)$ *or* $y=(x-5)(x+2)(12+3x)$

STEP-1: Put $f(x)=y=0$,get all values of x and plot them on number line. Here *values of* $x=-4,\ -2,\ 5$



STEP -2 :The number line will get divided into many sub-intervals.

Here intervals are $(-\infty,-4),(-4,-2),(-2,5)\ \ and\ (5,\infty)$

STEP -3 : Put +ve sign for $f(x)$ on the right side of right most number of the number line of STEP-1 and then alternately continue putting –ve sign,+ve sign for $f(x)$ up to the left side of left most number of the number line of STEP-1.

STEP -4 : Graph of $f(x)$ can also be drawn to discuss other nature of $f(x)$.(like increasing,decreasing,maximum,minimum,continuity etc.)

Note: (i) Remember in each linear factor of $f(x)$ coefficient of x is positive.

(ii) If in some linear factors of $f(x)$,coefficients of x are negative ,make them positive and after that if $f(x)$ $\text{become} - \text{f(x)}$.

Then for sign of $-f(x)$ just alter the process of STEP -3.

Example 2: If $f(x) = (2x+1)(7-x)$,

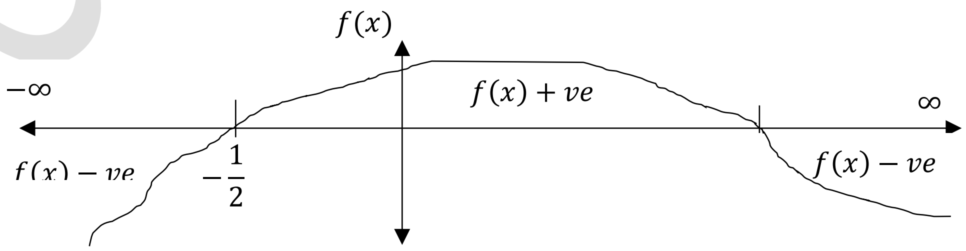
find values of x which makes f(x) positive or negative .

Sol: Here $f(x) = (2x+1)(7-x) = -(2x+1)(x-7)$

$\Rightarrow -f(x) = (2x+1)(x-7)$

Here f(x) negative in intervals are $\left(-\infty,-\frac{1}{2}\right), and\ (7,\infty)$

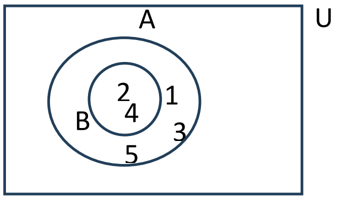
and f(x) positive in intervals are $\left(-\frac{1}{2},7\right)$



Venn -diagram: The diagrams used to represent sets are known as Venn diagrams .Usually area of circles are used to represent subsets of universal set and universal set is represented by area of rectangle containing all the circles. The elements belongs to a set are shown inside the circle, otherwise outside the circle.

Example: Represent the sets U=$\{1,2,3,4,\ldots\ldots10\}$,A=$\{1,2,3,4,5\}$,B=$\{2,4\}$using Venn diagram.

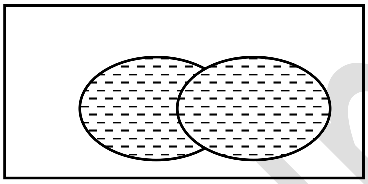
Sol^n:



Operations on Sets:

1. **Union of sets :** The union of two sets A and B (denoted by A∪ B) is the set of all elements which belongs to either A or B or both.

$\therefore$ A∪ $B = \{x/x \in A \text{ or } x \in B\}$



Example: If A=$\{1,2,3,4\}$, B=$\{2,3,7\}$,find A∪ B.

Sol^n: A∪ $B = \{1,2,3,4,7\}$

Properties of union:

(i) A∪ $B = B \cup A$
(ii) A∪ $\emptyset = A$
(iii) A∪ $A = A$
(iv) A∪ $U = U$,where U is the universal set.
(v) A∪ $(B \cup C) = (A \cup B) \cup C$
(vi) A⊂ $B \Rightarrow A \cup B = B$
(vii) A⊂ $A \cup B$ and B⊂ $A \cup B$

2. **Intersection of sets :** The intersection of two sets A and B (denoted by A∩ B) is the set of all elements which belongs to both A and B.

$\therefore$ A∩ $B = \{x/x \in A \text{ and } x \in B\}$

Example 1: If A=$\{1,2,3,4\}$, B=$\{2,3,7\}$,find A∩ B.

Sol^n: A$\cap B=\{2,3\}$

Example 2: If A=$\{1,3,5,7,\ldots..\}$, B=$\{2,4,6,8,\ldots\ldots\}$,find A$\cap B$.

Sol^n: A$\cap B=\emptyset$

Example 3: If A=$\{x/x=2n,n\in Z\}$B=$\{x/x=3n,n\in Z\}$,find A$\cap B$.

Sol^n: A=$\{\ldots.-4,-2,0,2,4,\ldots\ldots\}$,B=$\{\ldots-6,-3,-1,3,6,\ldots..\}$

$\therefore$ A$\cap B=\{\ldots.,-12,-6,0,6,12,\ldots.\}$

=$\{x/x=6n,n\in Z\}$

Properties of intersection :

(i) A$\cap B=\mathrm{B}\cap A$
(ii) A$\cap\emptyset=\emptyset$
(iii) A$\cap A=A$
(iv) A$\cap U=A$,where U is the universal set.
(v) A$\cap(\mathrm{B}\cap C)=(\mathrm{A}\cap B)\cap C$
(vi) A$\subset B\Rightarrow \mathrm{A}\cap B=A$
(vii) A$\cap B=\emptyset\Rightarrow A\ and\ B$ are disjoint sets.
(viii) $\mathrm{A}\cap B\subset A$ and $\mathrm{A}\cap B\subset B$

3.**Difference of two sets:** If A and B are two sets ,then the difference $A-B$ (or A\ B) is the set of all those elements of A which are not in B.

$\therefore$ A$-B=\{x/x\in A\ and\ x\notin B\}$.

$Similarly\ \mathrm{B}-A=\{x/x\in B\ and\ x\notin A\}$

Example**:** If A=$\{1,2,3,4\}$, B=$\{2,3,7\}$,find A$-B$ and $\mathrm{B}-A$.

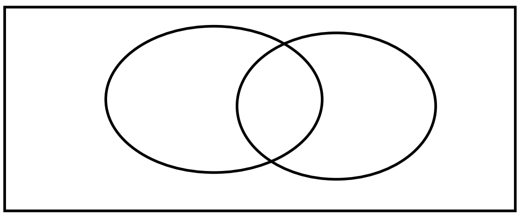
Sol^n: A$-B=\{1,4\}$ and $\mathrm{B}-A=\{7\}$.

Note: (i) A$-B\neq\mathrm{B}-A$

(ii) A$-\emptyset=A$ and $\emptyset-A=\emptyset$

4. **Symmetric difference of two sets:** The Symmetric difference of two sets A and B is denoted by A Δ B and is defined as A Δ B =(A$-B)\cup(\mathrm{B}-A)$

$=\{x\ /\ x\notin A\cap\mathrm{B}\}$



Example: If A={1,3,5,7,9}, B={2,3,5,7,11},find A Δ B and B Δ A .

Sol^n: $A \Delta B = (A - B) \cup (B - A)$

$=\{1,9\} \cup \{2,11\}=\{1,2,9,11\}$

$B \Delta A = (B - A) \cup (A - B)$

$= \{2,11\} \cup \{1,9\}=\{1,2,9,11\}$

Note: The sets $(A - B)$, $A\cap B$ and $(B - A)$ are mutually exclusive i.e intersection of any two of them is null set and their union is $A \cup B$.

5. **Complement of a set:** The complement of set A (denoted by A^c or A' or $\bar{A}$) is the set of all those elements in the universal set U which are not in A.

$\therefore A^c = U - A = \{x/x \in U \text{ and } x \notin A\}$

Example: If U={1,2,3,4,5,6,9}and A={2,3,5,7,11},find A^c.

Sol^n: $A^c = U - A = \{1,4,6,9\}$

Properties of complement :

(i) $A\cup A' = U$
(ii) $A\cap A' = \emptyset$
(iii) $(A')' = A$
(iv) $U' = \emptyset$
(v) $\emptyset' = U$

Laws of algebra of sets:

(i) Distributive laws: $A\cup (B\cap C)=(A \cup B) \cap (A \cup C)$

$A\cap (B\cup C)=(A \cap B) \cup (A \cap C)$

(ii) Associative laws: $A\cup (B\cup C)=(A \cup B) \cup C$

$A\cap (B\cap C)=(A \cap B) \cap C$

(iii) De-Morgan's laws: $(A \cup B)' = A' \cap B'$

$(A \cap B)' = A' \cup B'$

Important properties:

(i) $A - B = A \cap B'$
(ii) $A - (B \cup C) = (A - B) \cap (A - C)$
(iii) $A - (B \cap C) = (A - B) \cup (A - C)$
(iv) $A \cap (B - C) = (A \cap B) - (A \cap C)$

Theorem: If A and B are two finite sets ,then

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

Proof: Let $n(A) = m$ and $n(B) = n$

CaseI: Let A and B are disjoint ,then $A \cap B = \emptyset \Rightarrow n(A \cap B) = n(\emptyset) = 0$

$$\therefore\ n(A \cup B) = m + n = n(A) + n(B) - 0 = n(A) + n(B) - n(A \cap B)$$

CaseII: Let A and B are not disjoint ,then $A \cap B \neq \emptyset$

Let $n(A \cap B) = p$

$$\therefore\ n(A \cup B) = (m - p) + (n - p) + p$$

$$= m + n - p$$

$$= n(A) + n(B) - n(A \cap B)$$

Extension: $n(A \cup B \cup C) = n(A) + n(B) + n(C)$

$$-n(A \cap B) - n(B \cap C) - n(C \cap A) + n(A \cap B \cap C)$$

Note: If A,B,C are mutually exclusive ,then $n(A \cup B \cup C) = n(A) + n(B) + n(C)$

****

Chapter-1: Exercise

MCQ Type(1 mark each)

1. Let A and B be two non empty sets in the same universal set. Then A−B is equal to
   (a) A∩B (b)A′∩B (c) A∩B′ (d) none of these
2. If A={1,2,3,4,5}, then the number of proper subsets of A is
   (a) 120 (b) 30 (c) 31 (d) 32

3. If A={2,3}, B={4,5}, C={5,6}, then what is the number of elements in A x (B ∩ C)?**(2010-II)**

(a) 2 (b) 4 (c) 6 (d) 8

4. If set A and B are defined as $A = \{(x, y) \mid y = e^x, x \in R\}, B = \{(x, y) \mid y = x,\ x \in R\}$, then

(a)$B \subset A$ (b) $A \subset B$ (c)$A \cap B = \emptyset$ (d) $A \cup B = A$

5. Let N be the set of natural numbers and $A = \{n^2 | n \in N\}$ and $B = \{n^3 | n \in N\}$. Which of the following is correct?(2010-II)

(a)$A \cup B = N$ (b) $A \cap B$must be a finite set.

(c) The complement of $A \cup B$is an infinite set.

(d) $A \cap B$must be a proper subset of $\{m^6 | m \in N\}$.

6.If A, B, C are three sets, then what is $A - (B - C)$ equal to?(2008-II)

(a)$A - (B \cap C)$ (b) $(A - B) \cup C$

(c) $(A - B) \cup (A \cap C)$ (d) (A-B)∪ $(A - C)$

7. If $A = \{a, b, c, d\}$then what is number of proper subset of A?**(2010-I)**

(a) 16 (b) 15 (c) 14 (d) 12

8. If $n(A) = 115,\ n(B) = 326,\ \ n(A - B) = 47,$ then what is $n(A \cup B)$?

(a) 373 (b) 165 (c) 370 (d) 394

9. If $A \& B$ are two subset of a set X, then what is $A \cap (A \cup B)'$?**(2008-II)**

(a) A (b) B (c) $\emptyset$ (d) A'

10. If X & Y are any two non – empty sets then what is $(X - Y)'$ equals to?**(2009-I)**

(a) $X' - Y'$ (b) $X' \cap Y'$ (c) $X' \cup Y$ (c) $X - Y'$

11. If a set A contains 4 elements, then what is the number of elements in $A \times p(A)$?**(2008-II)**

(a) 16 (b) 32 (c) 64 (d) 128

12. If A & B are two sets, then $A \cap (A \cup B)$ is equal to

(a) A (b) B (c) $\emptyset$ (d) none of these

13. Let X be the universal set for sets A & B. If $n(A) = 200, n(B) = 300$ & $n(A \cap B) = 100$, then $n(A' \cap B')$ is equal to 300 provided $n(X)$ is equal to

(a) 600 (b) 700 (c) 800 (d) 900

14. If $n(A) = 4$ and, $n(B) = 7$ then the min^m and max^m value of $n\ (A \cup B)$respectively.

(a) 4, 11 (b) 4, 7 (c)7, 11 (d) none of these

15. If *A, B, C* are three sets and U is the universal set such that $n(U)=700$, $n(A)=200$, $n(B)=300$ and $n(A\cap B)=100$ then what is the value of $n(A'\cap B')$?

(a) 100 (b) 200 (c) 300 (d) 400

16. If *A*&*B* are two sets, then $(A-B)\cup(B-A)\cup(A\cap B)$is equal to

(a) $A\cup B$ (b) $A\cap B$ (c) A (d) B'

17.If A and B are two sets satisfying A-B = B-A, then which one of the following is correct? **(2007-II)**

(a) $A=\phi$ (b) $A\cap B=\phi$ (c) $A=B$ (d) none of these

18. If A and B are two subsets of set X, then what is the value of $A\cap(A\cup B)'$?

(a) A (b) B (c) ϕ (d) A'

19. If A & B are subsets of X, then what is $[A\cap(X-B)]\cup B$ equal to ?**(2009-I)**

(a) A$\cup$B (b) A$\cap$B (c) A (d) B

20. If A, B, C are three finite sets, then what is $[(A\cap B)\cap C]'$ equal to?

(a) $A'\cup B'\cap C'$ (b) $A'\cap B'\cap C'$ (c) $A'\cap B'\cup C'$ (d) $A\cap B\cap C$

21. If $A=\{0,1\}$ and $B=\{1,0\}$, then what is A x B equal to?

(a) $\{(0,1),(1,0)\}$ (b) $\{(0,0),(1,1)\}$ (c) $\{(0,1),(1,0),(1,1)\}$ (d) A x A

22. If $n(A)=43,\ n(B)=51\ \&\ n(A\cup B)=75$, then $n[(A-B)\cup(B-A)]$ is

(a) 53 (b) 45 (c) 56 (d) 66

23. If A & B are non-empty sets such that B$\subset$A, then

(a) $B'-A'=A-B$ (b) $B'-A'=B-A$ (c) $A'\cap B'=B-A$ (d) $A'\cup B'=A'-B'$

24. The set $A = \{x|x + 4 = 4\}$ can also be represented by**(2012-I)**

(a) 0 (b) $\emptyset$ (c) $\{\emptyset\}$ (d) $\{0\}$

25. If X and Y are two sets, then $X \cap (Y \cup X)'$ is equal to

(a) X (b) Y (c) $\emptyset$ (d) none

26. If $n(A) = 115, n(B) = 326, n(A - B) = 47$ then $n(A \cup B)$ is

(a) 373 (b) 165 (c) 370 (d) none

27. If $A \subseteq B$, then $B' - A'$ is equal to

(a) A' (b) B' (c) A–B (d) $\boldsymbol{\phi}$

28. If A=$\{(x, y)/ x^2 + y^2 = 25\}$ & B=$\{(x, y)/ x^2+9y^2=144\}$, then A⋂B contains

(a) one point (b) three points (c) two points (d) four point

29. Consider the following relations:-
 1. A–B = A – (A⋂B)
 2. A = (A⋂B)⋃(A–B)
 3. A – (B⋃C)=(A–B) ⋃(A–C)

Which of these is/are correct?

(a) (1) & (3) (b) (2) only (c)(2) & (3) (d) (1) & (2)

30. If A⊆B, then A △B is equal to

(a) (A–B)⋂(B–A) (b)A–B (c)B–A (d) None

31. If $\emptyset$ denotes the empty set, then which one of the following is correct?

(a) $\emptyset \in \emptyset$ (b) $\emptyset \in \{\emptyset\}$ (c) $\{\emptyset\} \in \{\emptyset\}$ (d) $0 \in \emptyset$

32. In a group of 500 students, there are 475 students who can speak Hindi & 200 can speak Bengali. What is the number of students who can speak Hindi only?

(a) 275 (b) 300 (c) 325 (d) 350

33. If A and B are two sets, then A∩(A∪B) equals

(a) A (b) B (c) $\emptyset$ (d) none

34. Two finite sets have m and n elements. The total number of subsets of the first set is 56 more than total number of subsets of second set. The values of m and n are **(2009-I)**

(a) 7,6 (b) 6,3 (c) 5,1 (d) 8,7

35. If X & Y are two sets, then $X \cap (Y \cup X)'$ equal to :

(a) X (b) Y (c) $\emptyset$ (d) None

36. If $n(A) = 4, n(B) = 3, n(A \times B \times C) = 24$, then $n(c)$ is

(a) 288 (b) 12 (c) 17 (d) 2

37. If A & B are non-empty sets such that $A \supset B$, then

(a) $B' - A' = A - B$ (b) $B' - A' = B - A$ (c) $A' - B' = A - B$ (d) $A' \cap B' = B - A$

38. In a class of 60 students, 45 students like music, 50 students like dancing, 5 students like neither. Then the number of students in the class who like both music and dancing is **(2015-I)**

(a) 35 (b) 40 (c) 50 (d) 55

39. Let S be the set of all distinct numbers of the form $\frac{p}{q}$, where p, q ∈{1,2,3,4,5,6,}. What is the cardinality of the set S?**(2016-II)**

(a) 21 (b) 23 (c) 32 (d) 36

40. If $A = \{(x, y)/x^2 + y^2 \leq 1, x, y\epsilon R\}$& B = $\{(x, y)/x^2 + y^2 \geq 4, x, y\epsilon R\}$ then

(a) $A - B = \emptyset$ (b) $B - A = \emptyset$ (c) $A \cap B \neq \emptyset$ (d)$A \cap B = \emptyset$

ASSERTION-REASON BASED QUESTIONS

In the following questions ,a statement of assertion (A) is followed by a statement of Reason (R) . Choose the correct answer out of the following choices .

(a) Both A and R are true and R is the correct explanation of A.

(b) Both A and R are true and R is not the correct explanation of A.

(c) A is true but R is false .

(d) A is false but R is true

1.Assertion (A):The interval $\{x: x \in R, -4 < x \leq 6\}$ is represented by $\{-4,6\}$.

Reason (R): The interval $\{x: x \in R, -12 < x < -10\}$ is represented by $(-12, -10)$

Ans: (d)

2. Assertion (A): The power set of the set $\{1,2\}$ is the set $\{\emptyset, \{1\}, \{2\}, \{1,2\}\}$

Reason (R): The power set is the set of all subsets of the set.

Ans: (a)

3.Assertion: $Let\ n(U) = 200$,$n(A) = 120$ and $n(A \cap B) = 30$ then n(A∩ $B') = 90$

Reason: n$(A - B) = n(A) - n(A \cap B)$

Ans: (a)

4. Assertion: If $A = \{x: x = 4n, n \in N\}$ and $B = \{x: x = 6n, n \in N\}$ then

$A \cap B = \{24,48,72,96, \dots.\}$

Reason: $A \cap B = \{ln: n \in N\ and\ l = LCM\ of\ (4,6)\}$

Ans: (d)

5. Assertion: If $A \cup B = A \cup C$ and $A \cap B = A \cap C$,then $B = C$

Reason: $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

Ans: (a)

6.Let $A = \{0,1,\{0,1\},2,3\}$ and $B = \{0,1\}$

Assertion:B is a subset of A.

Reason: B is an element of A

Ans: (b)

7. Assertion: $Let\ n(U) = 1000, n(S) = 720, n(T) = 450,$ then least value of n$(S \cap T)$ is 170

Reason:$n(S \cup T)$ is maximum when n$(S \cap T)$ is least.

Ans: (b)

Sol: $n(S \cup T) = n(S) + n(T) - n(S \cap T)$

$= 720 + 450 - n(S \cap T)$

$= 1170 - n(S \cap T) \leq n(U) \qquad (S \cup T \subset U)$

$\Rightarrow 1170 - \ n(S \cap T) \leq 1000$

$\Rightarrow n(S \cap T) \geq 170$

Least value is 170

SECTION-II(2 MARK EACH)

1. Using properties of set ,show that $(A - B) \cap B = \emptyset$
2. For any two sets prove that P(A∩B) = P(A)∩P(B).
3. For any two sets A and B, Prove that P(A) = P(B)⇒ A = B.
4. For any three sets A, B and C, Prove that A – (B – C) = (A – B)∪(A∩C).
5. Show that if $A \subset B, then\ C - B \subset C - A.$

Proof: Here $A \subset B$

Let $x \in C - B$

$\Rightarrow x \in C\ and\ x \notin B$

$\Rightarrow x \in C\ and\ x \notin A \quad [\because A \subset B]$

$\Rightarrow x \in C - A$

$\therefore C - B \subset C - A$

6.let A ,B ,C be the sets such that $A \cup B = A \cup C$ and $A \cap B = A \cap C$. Show that B=C.

Proof: Here $A \cup B = A \cup C \rightarrow (i)$

$A \cap B = A \cap C \rightarrow (ii)$

Now $B = B \cap (A \cup B)$

$= B \cap (A \cup C)$ (using (i))

$= (B \cap A) \cup (B \cap C)$ (using distributive law)

$= (A \cap B) \cup (B \cap C)$

$= (A \cap C) \cup (B \cap C)$ (using (ii))

$= (A \cup B) \cap C$ (using distributive law)

$= (A \cup C) \cap C$ (using (i))

$= C$

7. Assume that $P(A) = P(B)$ .Show that $A = B$

Proof: Let $x \in A$

$\Rightarrow \{x\} \subset A$

$\Rightarrow \{x\} \in P(A)$

$\Rightarrow \{x\} \in P(B)$ $[\because P(A) = P(B)]$

$\Rightarrow \{x\} \subset B$

$\Rightarrow x \in B$

$\Rightarrow A \subset B \rightarrow (i)$

Again, let $x \in B$

$\Rightarrow \{x\} \subset B$

$\Rightarrow \{x\} \in P(B)$

$\Rightarrow \{x\} \in P(A)$ $[\because P(A) = P(B)]$

$\Rightarrow \{x\} \subset A$

$\Rightarrow x \in A$

$\Rightarrow B \subset A \rightarrow (ii)$

From (i) and (ii)

$A = B$

8. For any sets A and B prove that $P(A \cap B) = P(A) \cap P(B)$

Proof: Let $X \in P(A \cap B)$

$\Rightarrow X \subset A \cap B$

$\Rightarrow X \subset A \; and \; X \subset B$

$\Rightarrow X \in P(A) \;\; and \; X \in P(B)$

$\Rightarrow X \in P(A) \cap P(B)$

$\Rightarrow \; P(A \cap B) \subset P(A) \cap P(B) \rightarrow (i)$

Again ,let $X \in P(A) \cap P(B)$

$\Rightarrow X \in P(A) \;\; and \; X \in P(B)$

$\Rightarrow X \subset A \; and \; X \subset B$

$\Rightarrow X \subset A \cap B$

$\Rightarrow X \in P(A \cap B)$

$\Rightarrow P(A) \cap P(B) \;\subset\; P(A \cap B) \rightarrow (ii)$

From (i) and (ii)

$P(A \cap B) = P(A) \cap P(B)$

9. In a group of 400 people in USA, 250 can speak Spanish and 200 can speak English. How many people can speak both Spanish and English?

SECTION-III(3 MARK EACH)

1.If A and B are two sets such that n(A)=17, n(B)=23 and n(A∪ B)=38 ,find the number of elements in exactly one of A and B.

ANS: $n(A \cup B) = 38$
=> $n(A) + n(B) - n(A \cap B) = 38$
=>$17 + 23 - \; n(A \cap B) = 38$
=>$n(A \cap B) = 40 - 38 = 2$
No of elements in A only = $n(A) - \; n(A \cap B) = 17 - 2 = 15$
No of elements in B only = $n(B) - \; n(A \cap B) = 23 - 2 = 21$
No of elements in exactly A and B=15+21=36

SECTION-IV(4 MARK EACH)

1. Out of 100 students, 15 passed in English, 12 passed in Mathematics, 8 in Science, 6 in English & Mathematics, 7 in Mathematics & Science, 4 in English & Science, 4 in all the three. Find how many passed in

   (a) English & Mathematics but not in Science

   (b) Mathematics only

   (c) more than one subject

2. There are 200 individuals with a skin disorder,120 had been exposed to chemical C_1,50 to chemical C_2 and 30 to both the chemicals C_1 and C_2 . Find the number of individuals exposed to

   (i) chemical C_1 but not chemical C_2
   (ii) chemical C_2 but not chemical C_1
   (iii) chemical C_1 or chemical C_2

3. A college awarded 38 medals in football, 15 in basketball and 20 in cricket. If these medals went to a total of 58 men and only three men got medals in all the three sports, how many received medals in exactly two of the three sports.

4. A market research group conducted a survey of 1000 consumers and reported that 720 consumers like product A and 450 consumers like product B, what is the least number that must have liked both products?

5. In a class of 150 students ,the following results were obtained in a certain examination .45 students failed in mathematics ,50 students failed in physics ,48 students failed in Chemistry ,30 students failed in both mathematics and physics ,32 students failed in physics and Chemistry.35 students failed in both mathematics and Chemistry ,25 students failed in all three subjects .Find

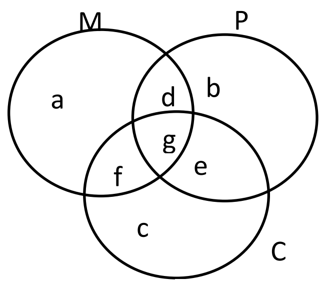
(i) The number of students failed in mathematics only.

(ii) The number of students failed in physics and Chemistry only.

Sol: Let M= set of students failed in mathematics.

P= set of students failed in physics.

C= set of students failed in chemistry.



As shown in Venn Diagram

$$a+d+f+g=45 \Rightarrow a=45-5-10-25=5$$

$$b+d+e+g=50 \Rightarrow b=50-5-7-25=8$$

$$c+e+f+g=48 \Rightarrow c=48-7-10-25=6$$

$$d+g=30 \Rightarrow d=5$$

$$e+g=32 \Rightarrow e=7$$

$$f+g=35 \Rightarrow f=10$$

$g = 25$

(i) Number of students failed in mathematics only $= a = 5$

(ii) Number of students failed in physics and chemistry only $= e = 7$

6. The students of a class are offered three languages(Hindi ,English and French).15 students learn all the three languages, whereas 28 students do not learn any language. The number of students learning Hindi and English but not French is twice the number of students learning Hindi and French but not English .The number of students learning English and French but not Hindi is thrice the number of students learning Hindi and French but not English .23 students learn only Hindi and 17 students learn only English. The total number of students learning French is 46 and the total number of students learning only French is 11.Find

(i) How many students learn precisely two languages.

(a) 55 (b) 40 (c) 30 (d) 13

(ii) How many students learn at least two languages.

(a) 15 (b) 30 (c) 45 (d) 55

(iii) How many students learn English and French.

(a) 30 (b) 43 (c) 45 (d) 73

(iv) What is the total strength of the class.

(a) 124 (b) 100 (c) 96 (d) 66

(v) How many students learn al least one language.

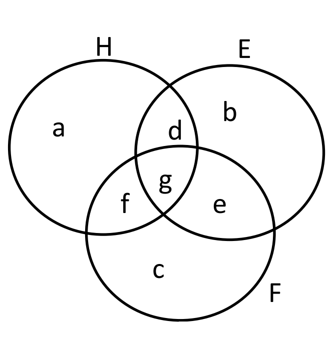
(a) 45 (b) 51 (c) 96 (d) none of these.

Sol: Let H= set of students learn Hindi.

E= set of students learn English.

F= set of students learn French.

As shown in Venn diagram



$g = 15, a = 23\ , b = 17, c = 11$

$c + e + f + g\ = 46$

$d\ = 2f$

$e = 3f$

$\therefore 11 + 3f + f + 15 = 46$

$\Rightarrow 4f = 40$

$\Rightarrow f = 5$

$\therefore e = 15, d = 10$

$(i) Number\ of\ students\ learn\ precisely\ two\ languages = d + e + f = 30.$

(ii) $Number\ of\ students\ learn\ at\ least\ two\ languages$

$$= d + e + f + g = 10 + 15 + 5 + 15 = 45$$

$(iii)\ Number\ of\ students\ learn\ English\ and\ French = e + g = 15 + 15 = 30$

(iv) Total strength of class= $a + b + c + d + e + f + g + 28 = 23 + 17 + 11 + 10 + 15 + 5 + 15 + 28 = 124$

(v) $Number\ of\ students\ learn\ at\ least\ one\ language$

$$= a + b + c + d + e + f + g = 96$$

7. In a group of students ,15 are taking algebra ,11 are taking biology ,9 are taking both algebra and biology and 3 are not taking either courses. Find

(i) how many students are taking only algebra?

(ii) how many students are taking only biology?

(iii) how many students are in the entire group?

8. In an examination 27% of the students failed in Maths and 31% failed in physics. If 6% students failed in both the subjects ,find the percentage of students

(i) failed in examination

(ii) passed in both the subjects.

9. In a group of 40 students ,22 are taking Maths ,18 are taking Physics ,14 are taking Chemistry,9 are taking Maths and Physics ,7 are taking Maths and Chemistry ,5 are taking Physics and Chemistry and 2 are taking all three subjects. How many students are not taking any of these subjects ?

10. In a group of students ,12 read Maths ,15 read Physics,11 read Chemistry ,4 read Maths only ,7 read Physics only ,3 read Physics and Chemistry only and 1 read Maths and Physics only .Find

(i) how many read all three subjects ?

(ii) how many read Maths and Chemistry only ?

(iii) how many read Chemistry only ?

(iv) how many students are there altogether ?

11. In a survey of 60 people ,it was found that 25 people read newspaper H, 26 read newspaper T, 26 read newspaper I, 9 read both H and I ,11 read both T and H ,8 read both T and I ,3 read all three newspaper .Find

(i) the number of people who read at least one of the newspaper .

(ii) the number of people who read exactly one newspaper.

12. A class has 175 students . The following description gives the number of students studying one or more of the subjects in this class :

Maths : 100 ,Physics : 70 ,Chemistry : 46 , Physics and Maths : 30 , Maths and Chemistry :28 , Physics and Chemistry : 23 , Maths ,Physics and Chemistry :18 . Find

(i) How many students are enrolled in Maths alone ,Physics alone and Chemistry alone.

(ii) the number of students who have not offered any of these subjects .

13. In a town of 10,000 families ,it is found that 40% families buy newspaper A ,20% families buy newspaper B and 10% families buy newspaper C . 5% families buy A and B , 3% buy B and C and 4% buy A and C . If 2% families buy all the newspapers ,find the number of families which buy

(i) A only (ii) B only (iii) none of A,B,C.

14. In a class of 120 students ,numbered 1 to 120 , all even numbered students opts for physics ,whose numbers are divisible by 5 opt for chemistry and those whose numbers are divisible by 7 opt for Maths. How many opt for none of these subjects ?

15. In a class of 50 students ,10 did not opt for Maths ,15 did not opt for science and 2 did not opt for either .How many students of the class opted for both Maths and science?

****

CHAPTER-2 RELATIONS AND FUNCTIONS

Cartesian Product of two sets:

Let A =set of a family members

$= \{father, mother, son\}$ and

B =set of food

$= \{milk, rice, chicken, fruits\}$

Let us make the all-possible pairs of food, the family member can eat.

The possible pairs are

(father, milk), (father, rice),(father, chicken),(father, fruits), (mother, milk),(mother, rice),(mother, chicken), (mother,fruits),(son,milk),(son,rice),(son,chicken),(son,fruits).

This set of ordered pairs is called cartesian product of set A and set B.

Definition:

Let A and B be two non-empty sets .Then the set of all ordered pairs (a, b), where $a \in A \text{ and } b \in B$ is called the cartesian product of A and B and is denoted by $A \times B$.

$\therefore A \times B = \{(a, b)/a \in A \text{ and } b \in B\}$

Example: Let $A = \{1,2,3\}$ and $B = \{4,5\}$,then

$A \times B = \{(1,4), (1,5), (2,4)(2,5), (3,4), (3,5)\}$

$B \times A = \{(4,1), (4,2), (4,3)(5,1), (5,2), (5,3)\}$

$A \times A = \{(1,1), (1,2), (1,3)(2,1), (2,2), (2,3), (3,1), (3,2), (3,3)\}$

$B \times B \times B = \{4,5\} \times \{4,5\} \times \{4,5\}$

$= \{(4,4), (4,5), (5,4), (5,5)\} \times \{4,5\}$

$= \{(4,4,4), (4,4,5), (4,5,4), (4,5,5), (5,4,4), (5,4,5), (5,5,4), (5,5,5)\}$

Note: (i) If $n(A) = p \text{ and } n(B) = q$, $\text{then } n(A \times B) = n(A)n(B) = pq$

(ii) $n[P(A \times B)] = 2^{pq}$

(iii) $(a, b) = (c, d) \Leftrightarrow a = c \text{ and } b = d$

(iv) If one of A or B is infinite then $A \times B$ is infinite.

(v) $A \times B \neq \mathrm{B} \times A$

(vi) $A \times B \times C = \{(a, b, c)/a \in A, b \in B, c \in C\}$

Some Important Results: For any three sets A, B, C

(i) $A \times (B \cup C) = (A \times B) \cup (A \times C)$

(ii) $A \times (B \cap C) = (A \times B) \cap (A \times C)$

(iii) $A \times (B - C) = (A \times B) - (A \times C)$

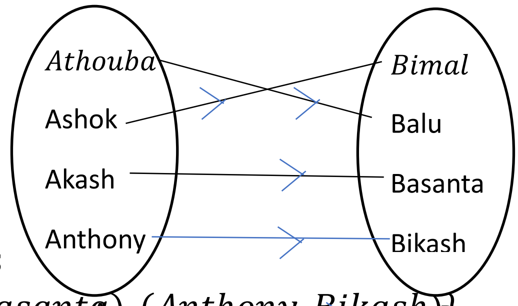
(iv) $(A \times B) \cap (C \times D) = (A \cap C) \times (B \cap D)$

(v) If $A \text{ and } B$ have n elements in common ,then $A \times B$ and $B \times A$ have n^2 elements in common.

Relation: The word relation used here, has the same usual meaning, which we have in our everyday life. By a relation we mean something like friendship, marriage, parenthood etc. “Is the friend of” , “Is the wife of”, “Is the father of” are the relations over the set of human beings .

Example 1: Let set $A = \{Athouba\ , Ashok, Akash, Anthony\}$ and set $B = \{Bimal\ \ , Balu\ , Basanta, Bikash\}$

Here if R is the relation "is friend of" from member of set, A to member of set B, then the above representation can be written as $Athouba\ R\ Balu$, $Ashok\ R\ Bimal,$ $Akash\ R\ Basanta,\ Anthony\ R\ Bikash$.



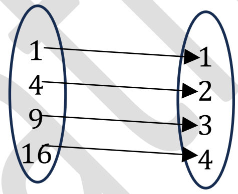
These relation can be written in the form of a set as

$R = \{(Athouba, Balu), (Ashok, Bimal), (Akash, Basanta), (Anthony, Bikash)\}$

$\quad = \{(x, y)/x \in A, y \in B\ and\ xRy\}$

Example 2: Let $A = \{1\ ,4\ 9, 16\}$ and $B = \{1\ ,2\ ,3,4\}$.

Here if R is the relation "is square of" from elements of set A to elements of set B. ,then $1R1,\ \ 4R2,\ \ 9R3, 16R4$.



This relation can be written in the form of a set as

$R = \{(1,1), (4,2), (9,3), (16,4)\}$

$\quad = \{(x, y)/x \in N, y \in Z, and\ x = y^2\}$

Relation: Let A and B are any two non-empty sets. Then any subset R of $A \times B$ (i.e. $\subseteq A \times B$) is called a relation from A to B .

Thus R is a relation from set A to set $B \ \Leftrightarrow R \subseteq A \times B$

$(a, b) \in R$ is also written as aRb and is read as $'a'$ is related to $'b'$ by the relation R and $(c, d) \notin R$ is also written as cRd and is read as $'c'$ is not related to $'d'$ by the relation R.

Example: Let $A = \{1,2,3\}$ and $B = \{a, b, c\}$,then

$$A \times B = \{(1, a), (1, b), (1, c), (2, a), (2, b), (2, c), (3, a), (3, b), (3, c)\}$$

(i) $R_1=\{(1, a), (2, b), (2, c)\}$ is a relation from A to B as $R_1 \subseteq A \times B$

(ii) $R_2=\{(a, 1), (c, 2), (b, 3)\}$ is a relation from B to A as $R_2 \subseteq B \times A$

(iii) $R_3=\{(1, a), (2, b), (1,1)\}$ is not a relation from A to B as $(1,1) \notin A \times B$

Domain of a relation R: If R is a relation from a non-empty set A to a non-empty set B ,then the set of all first component of all ordered pairs belonging to R is called the domain of relation R i.e. domain of R $(D_R) = \{x/x \in A\ and\ (x, y) \in R\ where\ y \in B\}$.

Range of a relation R: If R is a relation from a non-empty set A to a non-empty set B ,then the set of all second component of all ordered pairs belonging to R is called the range of relation R i.e. range of R $(R_R) = \{y/y \in B\ and\ (x, y) \in R\ where\ x \in A\}$.

Codomain of a relation R: If R is a relation from a non-empty set A to a non-empty set B ,then the set B is called the codomain of relation R.

Example : Let $A = \{1,2,3,5\}$ and $B = \{1,2,3,4,9\}$ .Let R= $\{(1,1),(2,4)(3,9)\}$ be the relation from A to B , then

$D_R = \{1,2,3\} \subseteq A$, $R_R = \{1,4,9\} \subseteq B$ and codomain= B

Note: (i) **Relation** from set A to set A is called relation on A.

(ii) If $n(A) = m$ and $n(B) = n$, then the total number of relations from A to B is 2^{mn}.

(iii) Total number of relations on A is 2^{m^2}

(iv) If $(a,b) \in R$,then a is called pre-image of b and b is called image of a under relation R.

(v) $D_R \subseteq A$ and $R_R \subseteq B$.

Example1: Let $A = \{2,4,6,8,10,12\}$, $B = \{1,2,3\}$. A relation R is defined from A to B by $R = \{(x,y)/x = 2y, x \in A, y \in B\}$. Write the relation R in roster form and find the domain and range of R.

Sol[n]: When $y = 1 \Rightarrow x = 2$

$y = 2 \Rightarrow x = 4$

$y = 3 \Rightarrow x = 6$

$\therefore$ R $= \{(2,1),(4,2),(6,3)\}$

$D_R = \{2,4,6\}$ and $R_R = \{1,2,3\}$

Example2: Let $A = \{1,2,3,4,5\}$, $B = \{1,4,9,16,25\}$. A relation R is defined from A to B by $R = \{(x,y)/y = x^2, x \in A, y \in B\}$. Write the relation R in roster form and find the domain and range of R.

Sol[n]: When $x = 1 \Rightarrow y = 1$

$x = 2 \Rightarrow y = 4$

$x = 3 \Rightarrow y = 9$

$x = 4 \Rightarrow y = 16$

$x = 5 \Rightarrow y = 25$

$\therefore$ R $= \{(1,1),(2,4),(3,9),(4,16),(5,25)\}$

$D_R = \{1,2,3,4,5\}$ and $R_R = \{1,4,9,16,25\}$

Example3: Find the domain and range of relation R given by

$$R = \left\{(x,y)/\ y = x + \frac{6}{x} \quad where\ x, y \in N\ \ and\ \ x < 6\right\}$$

Sol[n]: When $x = 1 \Rightarrow y = 7$

$x = 2 \Rightarrow y = 5$

$x = 3 \Rightarrow y = 5$

$x = 4 \Rightarrow y = 4 + \frac{6}{4} \notin N$

$x = 5 \Rightarrow y = 5 + \frac{6}{5} \notin N$

$\therefore$ R $= \{(1,7),(2,5),(3,5)\}$

$D_R = \{1,2,3\}$ and $R_R = \{7,5\}$

Example4: Let $R = \{(x,y)/x + 2y = 13 \quad where\ x, y \in N\}$ Find the domain and range of relation R .

Sol[n]: $x + 2y = 13 \Rightarrow y = \frac{13-x}{2}$

When $x = 1 \Rightarrow y = 6$

$x = 2 \Rightarrow y = \frac{11}{2} \notin N$

$x = 3 \Rightarrow y = 5$

$x = 4 \Rightarrow y = \frac{9}{2} \notin N$

$x = 5 \Rightarrow y = 4$

$x = 6 \Rightarrow y = \frac{7}{2} \notin N$

$x = 7 \Rightarrow y = 3$

$x = 8 \Rightarrow y = \frac{5}{2} \notin N$

$x = 9 \Rightarrow y = 2$

$x = 10 \Rightarrow y = \frac{3}{2} \notin N$

$x = 11 \Rightarrow y = 1$

$x = 12 \Rightarrow y = \frac{1}{2} \notin N$

$x = 13 \Rightarrow y = 0 \notin N$

$x = 14 \Rightarrow y = \frac{-1}{2} \notin N$

$\therefore$ R $= \{(1,6), (3,5), (5,4), (7,3), (9,2), (11,1)\}$

$D_R = \{1,3,5,7,9,11\}$ and $R_R = \{6,5,4,3,2,1\}$

Example 5: Let $A = \{1,2,3,4,5, \ldots \ldots 14\}$. Define a relation R on A by $R = \{(x, y)/3x - y = 0 \quad where\ x, y \in A\}$. Write down its domain ,range and codomain.

Sol^n: $3x - y = 0 \Rightarrow y = 3x$

When $x = 1 \Rightarrow y = 3$

$x = 2 \Rightarrow y = 6$

$x = 3 \Rightarrow y = 9$

$x = 4 \Rightarrow y = 12$

$x = 5 \Rightarrow y = 15 \notin A$

$\therefore$ R $= \{(1,3), (2,6), (3,9), (4,12)\}$

$D_R = \{1,2,3,4\}$, $R_R = \{3,6,9,12\}$ and codomain $= A$

Example 6 : Determine the domain and range of relation R defined by $R = \{(x, x + 5)/ \quad x \in \{0,1,2,3,4,5\}.\}$. Write down its domain ,range and codomain.

Sol^n: When $x = 0 \Rightarrow x + 5 = 5$

$x = 1 \Rightarrow x + 5 = 6$

$x = 2 \Rightarrow x + 5 = 7$

$x = 3 \Rightarrow x + 5 = 8$

$x = 4 \Rightarrow x + 5 = 9$

$x = 5 \Rightarrow x + 5 = 10$

$\therefore$ R = {(0,5), (1,6), (2,7), (3,8), (4,9), (5,10)}

$D_R = \{0,1,2,3,4,5\}$, $R_R = \{5,6,7,8,9,10\}$

Example 7 : Write the relation $R = \{(x, x^3) \ / x \ is \ prime \ number < 10\}$. In roster form.

Sol^n: When $x = 2 \Rightarrow x^3 = 8$

$x = 3 \Rightarrow x^3 = 27$

$x = 5 \Rightarrow x^3 = 125$

$x = 7 \Rightarrow x^3 = 343$

$\therefore$ R = {(2,8), (3,27), (5125), (7,343)}

Example 8: Let R be the relation on Z defined by

$R = \{(a, b) \ / \ a , b \in Z, a - b \ \ is \ an \ integer \ \}$. Find the domain and range of R.

Sol^n: We know $(a - b \ )is \ an \ integer \ \forall a, b \in Z$

$\therefore (a, b) \in R \ \ \forall a, b \in Z$

$D_R = Z$ and $R_R = Z$

Example 9: Let $A = \{1,2,3,5\}$ and $B = \{4,6,9\}$. Define a relation R from A to B by $R = \{(x, y) \ /difference \ between \ x \ and \ y \ is \ odd; x \in A, y \in B \ \}$

Write R in roster form.

Sol^n: Here $|x - y| = odd$

When $x = 1, y = 4 \Rightarrow |1 - 4| = 3 = odd$

$x = 1, y = 6 \Rightarrow |1 - 6| = 5 = odd$

$x = 1, y = 9 \Rightarrow |1 - 9| = 8 \neq odd$

$x = 2, y = 4 \Rightarrow |2 - 4| = 2 \neq odd$

$x = 2, y = 6 \Rightarrow |2 - 6| = 4 \neq odd$

$x = 2, y = 9 \Rightarrow |2 - 9| = 7 = odd$

$x = 3, y = 4 \Rightarrow |3 - 4| = 1 = odd$

$x = 3, y = 6 \Rightarrow |3 - 6| = 3 = odd$

$x = 3, y = 9 \Rightarrow |3 - 9| = 6 \neq odd$

$x = 5, y = 4 \Rightarrow |5 - 4| = 1 = odd$

$x = 5, y = 6 \Rightarrow |5 - 6| = 1 = odd$

$x = 5, y = 9 \Rightarrow |5 - 9| = 4 \neq odd$

$\therefore$ R = {(1,4), (1,6), (2,9), (3,4), (3,6), (5,4), (5,6)}

Example 10: Let $A = \{1,2,3,4,6\}$ .Let R be the relation on A defined by

$R = \{(a, b)/a \, , b \in A \, , b \ is \ exactly \ divisible \ by \ a\}$

(i) Write R in roster form.

(ii) Find the domain of R .

(iii) Find the range of R.

Sol^n: When $a = 1, b = 1 \Rightarrow b \ is \ divisible \ by \ a$

$a = 1, b = 2 \Rightarrow b \ is \ divisible \ by \ a$

$a = 1, b = 3 \Rightarrow b \ is \ divisible \ by \ a$

$a = 1, b = 4 \Rightarrow b\ is\ divisible\ by\ a$
$a = 1, b = 6 \Rightarrow b\ is\ divisible\ by\ a$
$a = 2, b = 2 \Rightarrow b\ is\ divisible\ by\ a$
$a = 2, b = 4 \Rightarrow b\ is\ divisible\ by\ a$
$a = 2, b = 6 \Rightarrow b\ is\ divisible\ by\ a$
$a = 3, b = 3 \Rightarrow b\ is\ divisible\ by\ a$
$a = 3, b = 6 \Rightarrow b\ is\ divisible\ by\ a$
$a = 4, b = 4 \Rightarrow b\ is\ divisible\ by\ a$
$a = 6, b = 6 \Rightarrow b\ is\ divisible\ by\ a$

(i) $R = \{(1,1), (1,2), (1,3), (1,4), (1,6), (2,2), (2,4), (2,6), (3,3), (3,6), (4,4), (6,6)\}$
(ii) Domain of R = {1,2,3,4,6}
(iii) Range of R = {1,2,3,4,6}

Example 11: Let R be a relation from Q to Q defined by
$R = \{(a, b) \ / \ a, b \in Q \ \ and\ a - b \in Z\}$ .Show that
$(i)(a, a) \in R \ \forall\ a \in Q$.
(ii) $(a, b) \in R$ *implies* that $(b, a) \in R$.
(iii) $(a, b) \in R$ and $(\mathrm{b, c}) \in \mathrm{R}$ implies that $(\mathrm{a, c}) \in \mathrm{R}$.

Proof: (i) Since $a - a = 0 \in Z \ \ \forall\ a \in Q$
$\Rightarrow (a, a) \in R \ \ \forall\ a \in Q$
(ii) Let $(a, b) \in R \Rightarrow a - b \in Z$
$\Rightarrow -(a - b) \in Z$
$\Rightarrow b - a \in Z$
$\Rightarrow (b, a) \in R$
$\therefore (a, b) \in R \Rightarrow (b, a) \in R$
(iii) Let $(a, b) \in R$ and $(\mathrm{b, c}) \in \mathrm{R}$
$\Rightarrow a - b \in Z \ \ and\ \ b - c \in Z$
$\Rightarrow (a - b) + (b - c) \in Z \ \ [\because sum\ of\ two\ integer\ is\ an\ integer]$
$\Rightarrow a - c \in Z$
$\Rightarrow (a, c) \in Z$
$\therefore (a, b) \in R$ and $(\mathrm{b, c}) \in \mathrm{R} \Rightarrow (\mathrm{a, c}) \in \mathrm{R}$.

All appointments are cadets but all cadets are not appointments.

Function(or mapping): Let A and B be two non-empty sets .Then the relation f $(i.e\ f \subseteq A \times B)$ is called a function (or mapping) from A to B if for every $x \in A$ there exist exactly one (unique) $y \in B$, where$(x, y) \in f$

The function f from A to B is denoted by $f: A \to B$ or $A \xrightarrow{f} B$

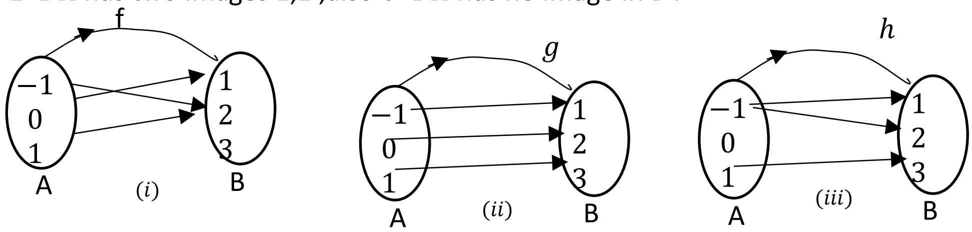
and $(x, y) \in f$ is written as $y = f(x)$,where y is called image of x under f (or value of f at x) and x is called the pre image of y under f .

Example: Let the two sets be $A = \{-1,0,1\}$ and $B = \{1,2,3\}$,then

(i) The relation $f = \{(-1,2), (0,1), (1,2)\}$ is a function from A to B , defined by $f(x) = x^2 + 1$ (called quadratic function)

(ii) The relation $g = \{(-1,1), (0,2), (1,3)\}$ is a function from A to B , defined by $g(x) = x + 2$ (called linear function)

(iii) The relation $h = \{(-1,1),(-1,2),(1,3)\}$ is not a function from A to B as $-1 \in A$ has two images 1,2 ,also $0 \in A$ has no image in B .



Note: If $f: A \rightarrow B$ then

(i) Domain of function $f = D_f$ = set A

(ii) Range of function $f = R_f$ = set of all f images of all $x \in A$

$= \{y \in B / f(x) = y \ \ \forall\, x \in A\} \subseteq B$

(iii) Co-domain of function f = set B

(iv) One or more elements of set A may have same f image in set B .

(v) There may be some elements in set B which may not have any pre-image in set A .

(vi) Every function is a relation but every relation is not a function.

(vii) If $n(A) = m$ and $n(B) = n$,then total number of distinct functions from A to B is n^m

Example: Let R be the set of real numbers and the relation f defined on R by $f = \left\{(x,y)/y = \frac{2x+1}{x-3}; x, y \in R\right\}$. Is the relation f a function, justify.

Sol^n: When $x = 3 \Rightarrow y = \frac{7}{0} \notin R$

For f to be a function ,every $x \in R$(domain set) there must be a unique image in R (codomain)

But $3 \in R(domain\ set)$ does not have any image in R (codomain)

$\therefore f$ is not a function.

Real valued function: A function $f: A \rightarrow B$ is said to be real valued function if $B \subseteq R$.

Real function: A function $f: A \rightarrow B$ is said to be real valued function if both $A, B \subseteq R$.

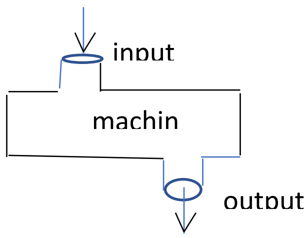
Domain of a function $f(x)$: Domain of $f(x)$ is the set of those values of x (also called input) for which $f(x)$ is defined (i.e real and unique, also called output).

Range of a function $f(x)$: Range of $f(x)$ is the set of all possible of $f(x)$ (i.e output) corresponding to the values of x in its domain.

Remark: Function is comparable with a machine used in daily life, in which for one input in the machine there must be a unique output.

For example ,if we put banana in juicer machine ,expected output is banana juice only.

Example:



function	domain
$f(x)=\frac{3}{x}$	$D_f=\{x\in R\ /\ x\neq 0\}$
$g(x)=\sqrt{x-6}$	$D_g=\{x\in R\ /\ x\geq 6\}$
$f(x)=\frac{2}{\sqrt{5-x}}$	$D_f=\{x\in R\ /\ x<5\}$
$f(x)=\sqrt{9-x^2}$	$D_f=\{x\in R\ /\ -3\leq x\leq 3\}$
$h(x)=\begin{cases}1, & x=\pi\\ 0, & x=3\end{cases}$	$D_h=\{\pi,3\}$

Example: Find the range of function $f(x)=x^2, x\in R$

Sol^n: Here $x\in R \Rightarrow x^2\geq 0$

∴ Range of $f(x)$ =[0,∞)

Working rules for finding domain of a function:

form of function	working rule for values of x
1. $\sqrt{f(x)}$	$f(x)\geq 0$,simplify it to get values of x.
2. $\frac{\text{constant}}{\sqrt{\text{f(x)}}}$	$f(x)>0$,simplify it to get values of x.
3. $\frac{\text{constant or g(x)}}{\text{f(x)}}$	$f(x)\neq 0$,simplify it to get values of x.
4.$f(x)\pm g(x)$	domain=dom f(x) ∩ dom g(x)
5. $f(x)\times g(x)$	domain=dom f(x) ∩ dom g(x)

Working rules for finding range of a function:

Step 1: Put $f(x)=y$

Step 2: Express x in terms of y ,say $x=g(y)$

Step 3: Find the values of y using the condition that the values of x are real and are withing the domain of $f(x)$ i.e values of y for which x exist.

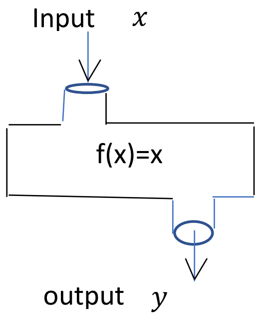
Note: Normally questions asked on functions are as follows:

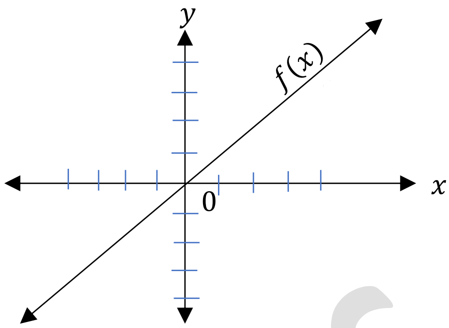
1. $f(x)$ given ,domain of $f(x)$ (i.e input) given ,find range (i.e output)
2. $f(x)$ given , find domain and range of $f(x)$.
3. Domain and range of $f(x)$ are given ,find $f(x)$.

Graph of a function $y=f(x)$: Graph of a function $y=f(x)$ is the figure obtained by joining the points (x,y), where x-coordinates are considered as inputs and y -coordinates as outputs.

Some standard real functions and their graphs:

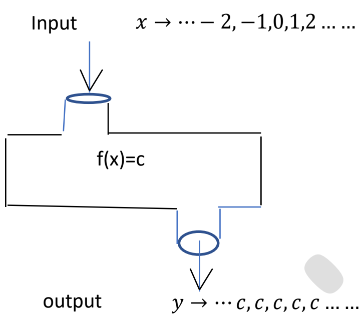
1. Identity function: A function $f: R \to R$ is said to be identity function if $f(x) = x \ \ \forall x \in R(domain\ set)$

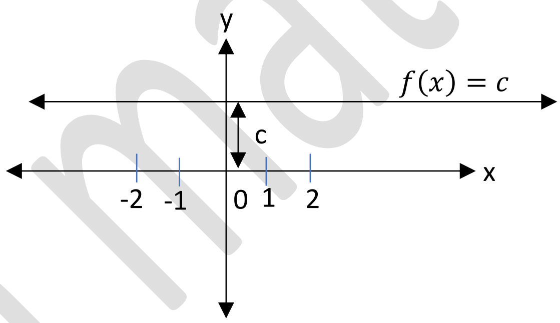




Here $D_f = (-\infty, \infty)$ and $R_f = (-\infty, \infty)$

2. **Constant function:** A function $f: R \to R$ is said to be constant function if $f(x) = c \ \ \forall x \in R(domain\ set), where\ c\ is\ a\ constant.$





Here $D_f = (-\infty, \infty)$ and $R_f = \{c\}$

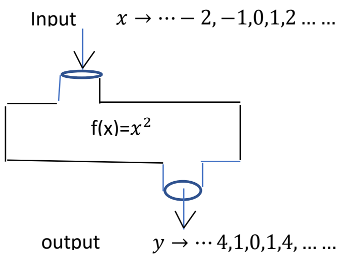
3. Polynomial function: A function $f: R \to R$ is said to be polynomial function if $f(x)$ is a polynomial in x i.e

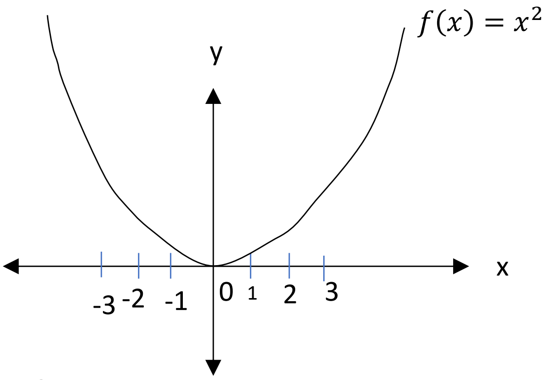
$f(x) = a_0 + a_1x + a_2x^2 + \cdots \ldots \ldots + a_nx^n$, where $n \in W, a_i \in R$

Example: $f(x) = 2 - 3x + 7x^2$ is a quadratic polynomial.

$f(x) = 1 - \sqrt{3}\,x + 5x^3$ is a cubic polynomial.

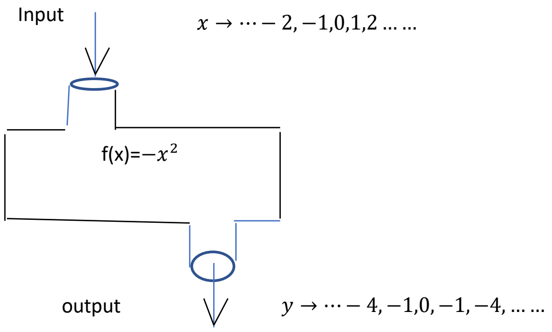
Graph of function $f: R \to R$,defined by $f(x) = x^2, \forall\, x \in R$.(quadratic function)

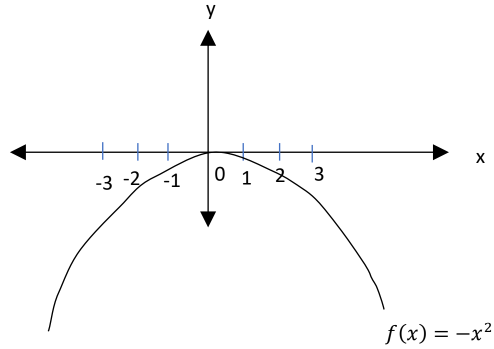




Here $D_f = (-\infty, \infty)$ and $R_f = [0, \infty)$

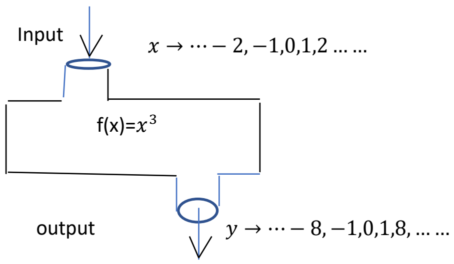
Graph of function $f: R \to R$,defined by $f(x) = -x^2, \forall\, x \in R$.(quadratic function)

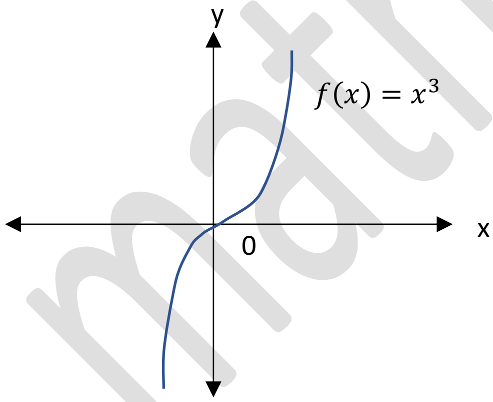




Here $D_f = (-\infty, \infty)$ and $R_f = (-\infty, 0]$

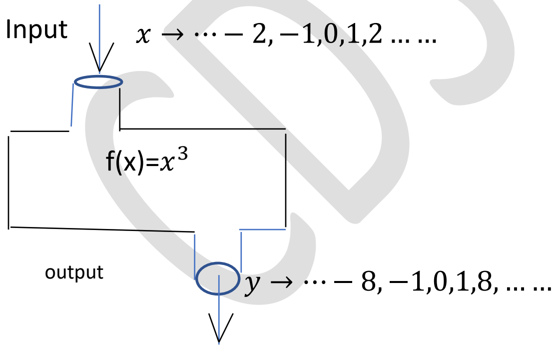
Graph of function $f: R \to R$,defined by $f(x) = x^3, \forall\, x \in R$.(cubic function)

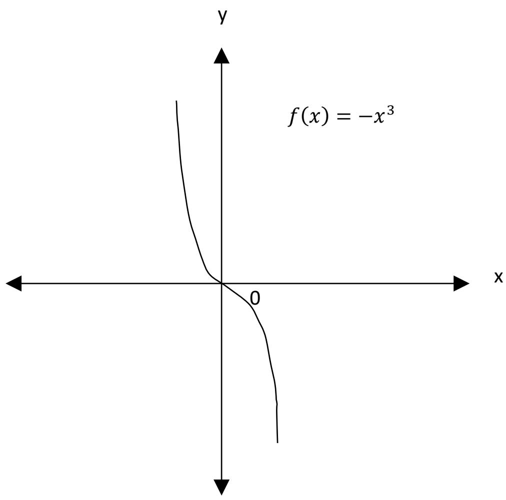




Here $D_f = (-\infty, \infty)$ and $R_f = (-\infty, \infty)$

Graph of function $f: R \to R$,defined by $f(x) = -x^3, \forall\, x \in R$.(cubic function)



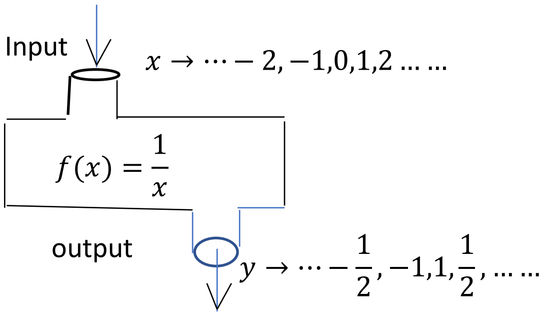


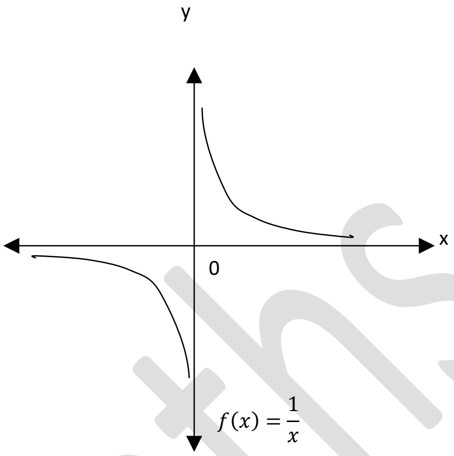
Here $D_f = (-\infty, \infty)$ and $R_f = (-\infty, \infty]$

4.Rational function: A function $f: R \to R$ is said to be a rational function if

$f(x) = \frac{p(x)}{q(x)}$, where $p(x)$ and $q(x)$ are polynomials and $q(x) \neq 0$,$\forall\, x \in R$.

Graph of function $f: R \to R$,defined by $f(x) = \frac{1}{x}, \forall\, x \in R\,, x \neq 0$.(rational function)



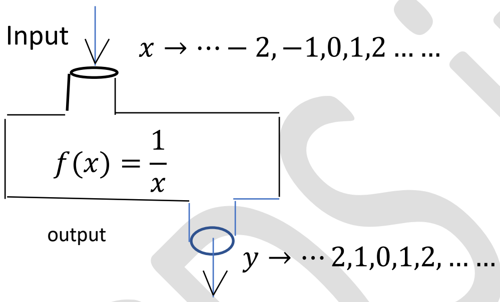


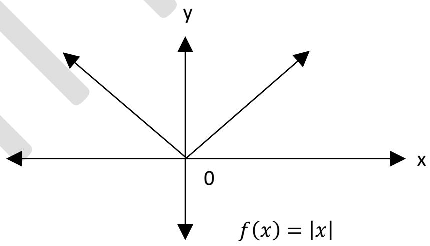
Here $D_f = R - \{0\}$ and $R_f = R - \{0\}$.

5.Modulus function: A function f: R → R defined by $f(x) = |x|, \forall\, x \in R$ is called the modulus function ,where

$$|x| = \begin{cases} x \;, when\; x > o \\ -x \;, when\; x < 0 \\ 0 \;, \quad when\; x = 0 \end{cases}$$

Graph of function $f: R \to R$,defined by $f(x) = |x|, \forall\, x \in R$.





Here $D_f = R$ and $R_f = (0, \infty)$

Properties of Modulus Functions:

(i) $|x| \leq \lambda \Rightarrow -\lambda \leq x \leq \lambda$

(ii) $|x| \geq \lambda \Rightarrow x \geq \lambda \;\; or \;\; x \leq -\lambda$

(iii) $x^2 \leq \lambda^2 \Rightarrow -\lambda \leq x \leq \lambda$

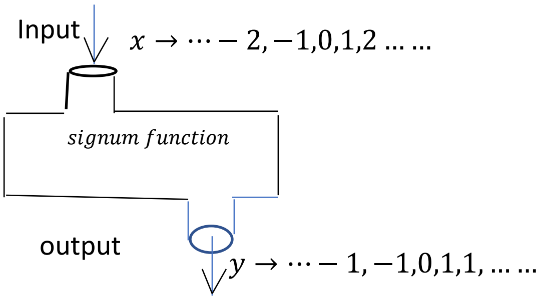
(iv) $x^2 \geq \lambda^2 \Rightarrow x \geq \lambda \;\; or\; x \leq -\lambda$ where $\lambda \geq 0$

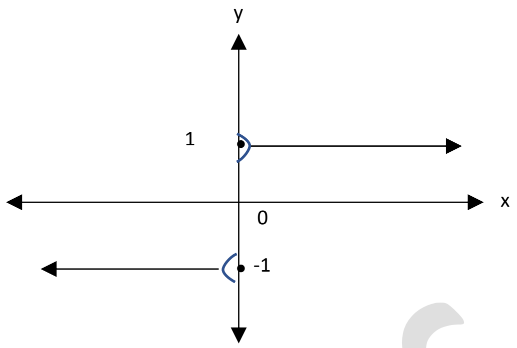
6. **Signum Function:** The function f: R → R defined by

$$f(\mathrm{x}) = \begin{cases} 1 \quad ,\text{when } \mathrm{x} > 0 \\ -1 \quad ,\text{when } \mathrm{x} < 0 \\ 0 \quad ,\text{when } \mathrm{x} = 0 \end{cases}$$

is called the signum function.

Graph of signum function:





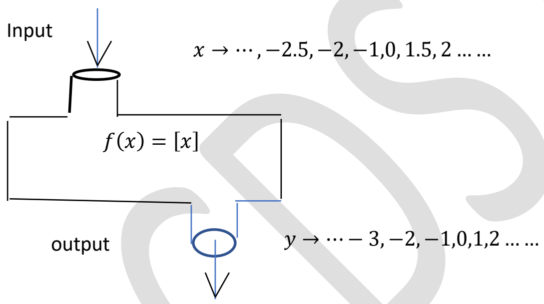
Here $D_f = R$ and $R_f = \{-1, 0, 1\}$

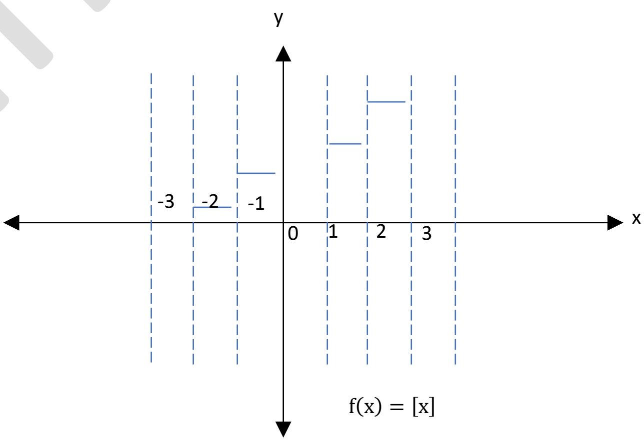
7.Greatest integer function (GIF) (or step function): The function f: R → R defined by $f(x) = [x]\ \forall\ x \in R$ is called GIF ,where

$$[x] = \begin{cases} x \text{, if x is an integer} \\ \text{greatest integer} < x \text{, if x is not an integer} \end{cases}$$

Example : $[3] = 3$, $[-7] = -7$, $[0] = 0$, $[.5] = 0$, $[-2.3] = -3$ etc.

Graph of function $f: R \to R$,defined by $f(x) = [x], \forall\ x \in R$.





Here $D_f = R$ and $R_f = Z$

Properties of GIF.

(i) $x - 1 < [x] \le x$

(ii) $[x \pm k] = [x] \pm k$, $k \in Z$

(iii) $[x] + [-x] = \begin{cases} 0, if\ x \in Z \\ -1, if x \notin Z \end{cases}$

(iv) $x - [x] = \begin{cases} 0, if\ x \in Z \\ (0,1), if\ x \notin Z \end{cases}$

(v) $[-x] = -[x]$

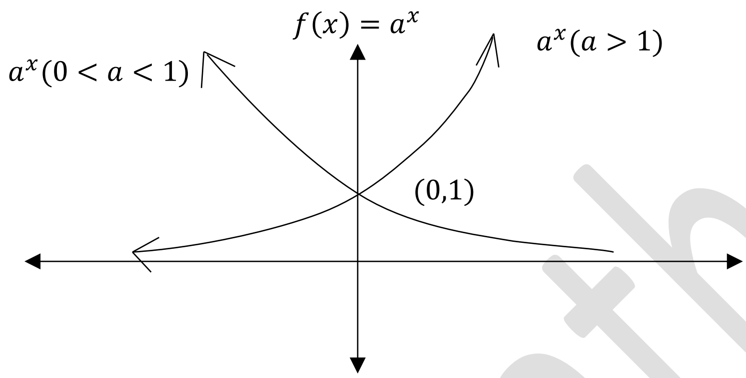
(vi) $[x] = k \Rightarrow k \le x < k+1\,, k \in Z$

(vii) $[x] \ge 5 \Rightarrow x \ge 5$

(viii) $[x] \le 5 \Rightarrow x < 6$

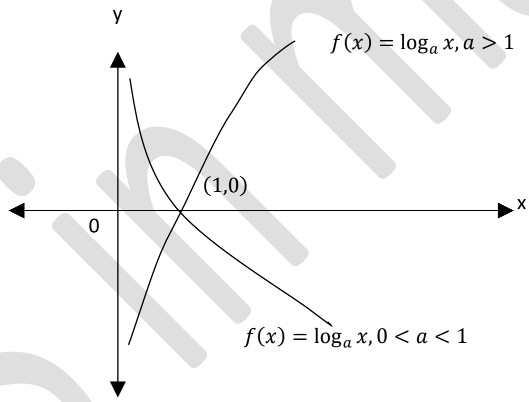
8. **Exponential Function:** The function $f: R \to R$ defined by

$f(x) = a^x \quad \forall\, x \in R$, (where $a > 0, a \ne 1$) is called exponential function.



Here $D_f = R$ and $R_f = (0, \infty)$

9. **Logarithmic Function:** The function $f(x) = \log_a x$ (where $a > 0, a \ne 1, x > 0$) is called logarithmic function.



Here $D_f = (0, \infty)$ and $R_f = (-\infty, \infty)$

Algebra of Real Functions : If $f: X \to R$ and $g: X \to R$ are any two real functions ,where $X \subset R$, then

(i) $(f+g): X \to R$ is defined as $(f+g)(x) = f(x) + g(x)$,$\forall x \in X$

(ii) $(f-g): X \to R$ is defined as $(f-g)(x) = f(x) - g(x)$, $\forall x \in X$

(iii) $(\lambda f): X \to R$ is defined as $(\lambda f)(x) = \lambda f(x)$, $\forall x \in X$ where λ is a scalar (i.e. real number).

(iv) $(fg): X \to R$ is defined as $(fg)(x) = f(x)g(x)\ \forall x \in X$

(v) $\left(\frac{f}{g}\right): X \to R$ is defined as $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$,if $g(x) \ne 0$, $\forall x \in X$

Note: If $f: X \to R$ and $g: Y \to R$ are any two real functions ,where $X, Y \subset R$, then $f+g,\ f-g,\ fg,\ \frac{f}{g}$ are to be defined on the set $(X \cap Y)$.

(i) $(f+g): X \cap Y \to R$ is defined as $(f+g)(x) = f(x) + g(x)$,$\forall x \in X \cap Y$

(ii) $(f-g): X \cap Y \to R$ is defined as $(f-g)(x) = f(x) - g(x), \forall x \in X \cap Y$

(iii) $(fg): X \cap Y \rightarrow R$ is defined as $(fg)(x) = f(x)g(x) \ , \forall x \in X \cap Y$

(iv) $\left(\frac{f}{g}\right): X \cap Y - \{x/g(x) = 0\} \rightarrow R$ is defined as $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} \ , \ \forall x \in X \cap Y$

(v) If $f: X \rightarrow R$ then $\left(\frac{1}{f}\right): X - \{x/f(x) = 0\} \rightarrow R$ defined by $\left(\frac{1}{f}\right)(x) = \frac{1}{f(x)}$

Example 1: Let $f(x) = \sqrt{x}$ and $g(x) = x$ be two functions defined over the set of non-negative real numbers .

Find $(f+g)(x)$, $(f-g)(x)$, $(fg)(x)$, $\left(\frac{f}{g}\right)(x)$

Sol[n]: $(f+g)(x) = f(x) + g(x) = \sqrt{x} + x$

$$(f-g)(x) = f(x) - g(x) = \sqrt{x} - x$$

$$(fg)(x) = f(x)g(x) = \sqrt{x}(x) = x^{\frac{3}{2}}$$

$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} = \frac{\sqrt{x}}{x} = x^{-\frac{1}{2}} \ , x \neq 0$$

Example2: Let $f = \{(1,1), (2,3), (0,-1), (-1,-3)\}$ be a function from Zto Z defined by $f(x) = ax + b$,for some integers a, b .Determine a, b.

Sol[n]: $(1,1) \in f \Rightarrow f(1) = 1 \Rightarrow a + b = 1$

$$(0,-1) \in f \Rightarrow f(0) = -1 \Rightarrow 0 + b = -1 \Rightarrow b = -1$$

$\therefore a = 2, b = -1$

Example3: Find the domain and range of the following real functions:

(i) $f(x) = 2 - 3x \quad , x > 0$ [linear function]

(ii) $f(x) = \sqrt{x^2 - 5}$

(iii) $f(x) = \frac{1}{\sqrt{x^2+36}}$

(iv) $f(x) = x^2 + x + 1$ [quadratic function]

(v) $f(x) = \frac{5x+1}{x+3}$ $\left[\frac{linear\ function}{linear function}\right]$

(vi) $f(x) = \frac{x^2+5}{x^2+1}$ $\left[\frac{quadratic\ function}{quadratic\ function}\right]$

(vii) $f(x) = \sqrt{x - 1}$

(viii) $f(x) = \frac{1}{\sqrt{2x-1}}$

(ix) $f(x) = \frac{x}{x^2+3}$ $\left[\frac{linear\ function}{quadratic\ function}\right]$

Sol[n]:(i) For domain:

$\therefore D_f = \{x \ / \ x > 0\}$

For range: $y = 2 - 3x \Rightarrow x = \frac{2-y}{3}$

Since $x > 0 \Rightarrow \frac{2-y}{3} > 0 \Rightarrow 2 - y > 0 \Rightarrow y < 2$

$\therefore R_f = (-\infty, 2)$

Sol[n]:(ii) For domain:

$x^2 - 5 \geq 0$

$\Rightarrow x^2 \geq 5$
$\Rightarrow x \geq \sqrt{5}\ or\ x \leq -\sqrt{5}$
$\therefore D_f = \left(-\infty, -\sqrt{5}\right] \cup [\sqrt{5}, \infty)$
For range: $y = \sqrt{x^2 - 5}$ (y$\geq$ 0)
$\Rightarrow y^2 = x^2 - 5$
$\Rightarrow x^2 = y^2 + 5, \Rightarrow x = \pm\sqrt{y^2 + 5}$
Since x $\in R \Rightarrow \sqrt{y^2 + 5} \in R$
$\Rightarrow y^2 + 5 \geq 0$
$\Rightarrow y^2 \geq -5$
But y $\geq$ 0
$\therefore R_f = [0, \infty)$
Sol[n]:(iii)
For domain:
$x^2 + 36 > 0 \Rightarrow x \in (-\infty, \infty)$
$\therefore D_f = R$
For range:
$y = \frac{1}{\sqrt{x^2+36}}$ (y> 0)
$\Rightarrow y^2 = \dfrac{1}{x^2 + 36}$
$\Rightarrow x^2 + 36 = \frac{1}{y^2}, \Rightarrow x^2 = \frac{1}{y^2} - 36$
$Now\ x^2 \geq 0$
$\Rightarrow \dfrac{1}{y^2} - 36 \geq 0$
$\Rightarrow \frac{1}{y^2} \geq 36, \Rightarrow y^2 \leq \frac{1}{36}, \Rightarrow -\frac{1}{6} \leq y \leq \frac{1}{6}$
$\therefore R_f = (0, \dfrac{1}{6}]$

Sol[n]:(iv) For domain:
$\therefore D_f = R$
For range:
$y = x^2 + x + 1, \Rightarrow x^2 + x + (1 - y) = 0$
$Since\ \ x \in R$
$\Rightarrow b^2 - 4ac \geq 0, \Rightarrow 1 - 4(1 - y) \geq 0$
$\Rightarrow 4y - 3 \geq 0, \Rightarrow y \geq \frac{3}{4}$
$\therefore R_f = [\dfrac{3}{4}, \infty)$
Sol[n]:(v) For domain:

$x + 3 \neq 0 \Rightarrow x \neq -3$

$\therefore D_f = \{x \in R / x \neq -3\}$

For range:

$y = \dfrac{5x+1}{x+3}$

$\Rightarrow xy + 3y = 5x + 1$

$\Rightarrow x(y-5) = 1 - 3y$

$\Rightarrow x = \dfrac{1-3y}{y-5}$

But $y - 5 \neq 0 \Rightarrow y \neq 5$

$\therefore R_f = R - \{5\}$

Sol^n:(vi) For domain:

$D_f = R$

For range:

$y = \frac{x^2+5}{x^2+1}$

$\Rightarrow x^2y + y = x^2 + 5$

$\Rightarrow (y-1)x^2 + y - 5 = 0$

$Since \;\; x \in R$

$\Rightarrow b^2 - 4ac \geq 0$

$\Rightarrow 0 - 4(y-1)(y-5) \geq 0$

$\Rightarrow 4(y-1)(y-5) \leq 0$

$\Rightarrow (y-1)(y-5) \leq 0$

$\Rightarrow 1 \leq y \leq 5$

$\therefore R_f = [1,5]$

Sol^n:(vii) For domain:

$x - 1 \geq 0 \Rightarrow x \geq 1$

$\therefore D_f = [1, \infty)$

For range:

$y = \sqrt{x-1}$ (y$\geq$ 0)

$\Rightarrow x - 1 = y^2, \Rightarrow x = 1 + y^2$

$Since \;\; x \geq 1$

$\Rightarrow \;\; 1 + y^2 \geq 1 \;\; , \Rightarrow \;\; y^2 \geq 0$

$\Rightarrow y \geq 0$

$\therefore R_f = [0, \infty)$

Sol^n:(ix) For domain:

$x^2 + 3 \neq 0 \Rightarrow x^2 \neq -3$

$\therefore D_f = R$

For range:

$$y = \frac{x}{x^2+3}$$
$$\Rightarrow yx^2 - x + 3y = 0 \qquad (y \neq 0)$$
$$Since\ \ x \in R$$
$$\Rightarrow b^2 - 4ac \geq 0$$
$$\Rightarrow 1 - 4y.3y \geq 0$$
$$\Rightarrow 2y^2 \leq 1$$
$$\Rightarrow y^2 \leq \frac{1}{12}$$
$$\Rightarrow -\frac{1}{2\sqrt{3}} \leq y \leq \frac{1}{2\sqrt{3}}$$
$$\therefore R_f = \left[-\frac{1}{2\sqrt{3}}, \frac{1}{2\sqrt{3}}\right] - \{0\}.$$

****

Chapter-2 :Exercise

MCQ Type

1. If A={1,2,3}, B={1,4,6,9} and R is a relation from A to B defined by 'x is greater than y', then the range of R is
   (a) {1,4,6,9} (b) {4,6,9} (c) {1} (d) none of these

2. Let $f: R \to R$ defined by $f(x) = \frac{|x|}{x}, x \neq 0, f(0) = 2$. what is the range of f?(2009-II)

(a){1,2} (b) {-1,1) (c) {-1,1,2} (d) {1}

3.If $f(x) = (a - x^n)^{\frac{1}{n}}$, then $f(f(x))$ is equal to

(a)x (b) $a - x$ (c)x^n (d) $x^{\frac{-1}{n}}$

4. Let $f\left(x + \frac{1}{x}\right) = x^2 + \frac{1}{x^2}, x \in R - \{0\}$, then $f(x)$ is equal to

(a) x^2 (b) $x^2 - 1$ (c) $x^2 - 2$, when $|x| \geq 2$ (d) none

5. The range of function $f(x) = |x|$ is
   (a) (0,α) (b) (−α,0) (c) [0,α) (d) none
6. Let R be a relation in N defined by R={$(x, y)/x$+2y=8}. The range of R is
   (a) {2,4,6} (b) {1,2,3} (c) {1,2,3,4,6} (d) none

For the next three (03) items that follow:-**(2014-II)**
Consider the function $f(x) = \frac{x-1}{x+1}$

7. What is $\frac{f(x)+1}{f(x)-1} + x$ equal to?
   (a) 0 (b) 1 (c) 2 x (d) 4 x

8. What is $f(2x)$ equal to?
   (a) $\frac{f(x)+1}{f(x)+3}$ (b) $\frac{f(x)+1}{3f(x)+1}$ (c) $\frac{3f(x)+1}{f(x)+3}$ (d) $\frac{f(x)+3}{3f(x)+1}$

9. What is $f(f(x))$ equal to?
   (a) x (b)$-x$ (c) $-\frac{1}{x}$ (d) none

10. If $R = \{(x, y)/x, y \in Z, x^2 + y^2 \leq 4\}$ is a relation in Z, then domain of R is:
(a) {0,1,2} (b) {0,-1,-2} (c) {-2,-1,0,1,2} (d) None

11. The range of the function $f(x) = \frac{1+x^2}{x^2}$ is equal to
(a)$[0,1)$ (b) $(0,1)$ (c) $(1, \alpha)$ (d) $[1, \alpha)$

12. If $f(x) = x^2 - x^{-2}$, $x \epsilon R - \{0\}$, then $f\left(\frac{1}{x}\right)$ is equal to
(a) $f(x)$ (b) $-f(x)$ (c) $\frac{1}{f(x)}$ (d) $(f(x))^2$

13. Let R be a relation in N defined by $R = \{(x, y)/2x + y = 8\}$, then, range of R is
(a) {1,2,3} (b) {2,4,6} (c) {1,2,3,4,6} (d) none

ASSERTION-REASON BASED QUESTIONS

In the following questions ,a statement of assertion (A) is followed by a statement of Reason (R) . Choose the correct answer ou of the following choices .
(a) Both A and R are true and R is the correct explanation of A.
(b) Both A and R are true and R is not the correct explanation of A.
(c) A is true but R is false .
(d) A is false but R is true .

1 .Assertion: Number of relations from the set $A = \{1,3\}$ to the set $B = \{-1,0,1\}$ is 32.
Reason : Number of relation from a set A to another set B is $2^{n(A\times B)}$
Ans: (d)

2. Assertion: If $(x + y, 3) = (5, x - y)$ then $x = 4, y = 1$
Reason :Two ordered pairs are equal if and only if their corresponding elements are equal.
Ans: (a)

3.Let $A = \{2,3,4,5\}$ and a relation R on A is defined as $R = \{(a, b): a\ is\ divisible\ b\ and\ a, b \in A\}$
Assertion: R in roster form is $\{(2,2), (2,4), (3,3), (4,4), (5,5)\}$
Reason : Domain and range of R is A.
Ans: (d)

4. Assertion: A relation $R = \{(1,3), (2,2), (3,1)\}$ defined on the set $A = \{1,2,3\}$ is a function
Reason : A relation from a set A to another set B is said to be a function if every elements of A is related to a unique element of B.
Ans: (a)

5. Assertion: For two sets A and B with $n(A) = 2$ and $n(B) = 2$,if$(a, 1), (b, 1), (a, 2) \in A \times B$, then $A = \{a, b, c\}$ and $B = \{1,2\}$
Reason :For two non-empty sets A and B $A \times B = \{(x, y): x \in A\ and\ y \in B\}$
Ans: (d)

SECTION-II(2 MARK EACH)

1.Let $f = \{(1,1), (2,3), (0, -1), (-1, -3)\}$ be a linear function from Z to Z .Find $f(x)$.

2.Let f be the subset of Z× Z defined by $f = \{(ab, a + b)/a, b \in Z\}$Is f a function from Z to Z ? Justify your answer.

Sol^n.: $f = \{(ab, a + b)/a, b \in Z\}$

$f(0 \times 0) = 0 + 0 = 0$ => $f(0) = 0$

$f(0 \times 1) = 0 + 1 = 1$ => $f(0) = 1$

Since $0 \in Z$ has more than one image and so f is not a function.

3. If P=$\{x / x < 3, x \in N\}, Q = \{x / x \leq 2, x \in W\}$ then find $(P \cup Q) \times (P \cap Q)$

ANS: $P = \{x/x < 3, x \in N\}$ =$\{1,2\}$

$Q = \{x/ x \leq 2, x \in W\} = \{0,1,2\}$

$(P \cup Q) = \{0,1,2\}$

$(P \cap Q) = \{1,2\}$

$(P \cup Q) \times (P \cap Q) = \{(0,1)(0,2)(1,1)(1,2)(2,1)(2,2)\}$

4. If 'f' is a real function defined by $f(x) = \frac{x-1}{x+1}$, then Prove that $f(2x) = \frac{3f(x)+1}{f(x)+3}$.

5. If $y = f(x) = \frac{ab-ax}{a-bx}$, Show that x = f(y).

6. Find the domain of the function: $f(x) = \frac{x^2+2x+1}{x^2-8x+12}$

7. Let R be a relation on N defined by $R = \{(1+x, 1+x^2)/\ x \preccurlyeq 4, x \in N\}$

   Find domain & range of R.

8. Let $3f(x) + f\left(\frac{1}{x}\right) = \frac{1}{x} + 1$,then find $f(x)$ **(NDA-I 2024)**

SECTION-III(3 MARK EACH)

1. Let R be a relation from Q to Q defined by $R = \{(a,b)/\ a,\ b \in Q\ and\ a-b \in z\}$ show that
   (i) (a, a) ∈ R for all a ∈ Q
   (ii) (a, b) ∈ R implies that (b, a) ∈ R
   (iii) (a, b) ∈ R and (b, c) ∈ R implies that (a, c) ∈ R.
2. Let 'R' be the relation in the set Z of all integers defined by (a,b) ∈ R ⇒(a-b) is divisible by 2.
   Prove that
   (i) (a,a) ∈ R for all a ∈ Z
   (ii) (a,b) ∈ R ⟹(b,a) ∈ R for all a,b∈ Z
   (iii) (a,b) ∈ R and (b,c) ∈ R ⇒(a,c) ∈ R for all a,b,c ∈ Z
3. Find the domain and range of the real function $f(x) = \sqrt{9-x^2}$
4. Let R={(x,y)|x,y∈R, y=2x+8}, if (a-2) and $(4, b^2)$ ∈ R, find values of a and b.
5. Find the relation R on Z defined by {(a,b)|a,b∈Z and |x| = |y|}. Also write its domain and range.

6. Let $X = \{2,3,4,5\}$ and $Y = \{7,9,11,13,15,17\}$. $Define\ a\ relation\ f\ from\ X\ to\ Y\ by$:
$$f = \{(x,y) : x \in X, y \in Y\ and\ y = 2x + 3\}.$$

(i)Write f in roster form.

(ii)Find domain of f and range of f

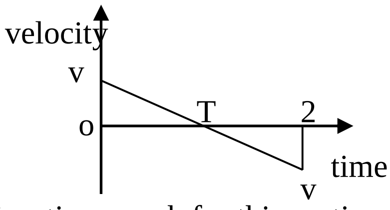
(iii)Show that f is a function from $X\ to\ Y$.

****

SAINIK SCHOOL IMPHAL
SUMMER BREAK ASIGNMENT 2025-26
SUB: PHYSICS
CLASS: 11

	UNIT & DIMENSION:
1.	Which one of the following pairs does not have the same dimensions? (a) Potential energy and kinetic energy (b) Density and specific gravity (c) Focal length and height (d) Gravitational force and frictional force
2.	The value of which one of the following quantities remains same in all systems of units? (a) Acceleration due to gravity (b) Specific gravity (c) Pressure (d) Density
3.	Which one of the following physical quantity has the same unit as that of pressure ? (a) Angular momentum (b) Stress (c) Strain (d) Work
4.	What do you meant by unit of a physical quantity?
5.	What are the advantages of SI Unit over other system of units?
6.	A physical quantity 'X' is measured with reference to the units 'A' and 'B', where A > B. In which unit 'X' will have higher numerical value? Explain your answer.
7.	We have a relation between four different quantities as $W + X = Y + Z$. What can you say about the units of these quantities?
8.	In a Vernier calliper, N divisions of vernier scale coincide with (N-1) divisions of main scale (in which one division represents 1 mm). Find the least count of the instrument (in cm).
9.	In a slide calliper, (M+1) number of vernier divisions is equal to M number of smallest main scale divisions. If d units is the magnitude of the smallest main scale division, then find the magnitude of the vernier constant.
10.	The length and breadth of a metal sheet are 3.124 m and 3.02 m respectively. Calculate the area of this sheet up to four correct significant figure.
11.	Find the dimensions of $a \times b$ in the relation $P = \frac{a-t^2}{b\sqrt{x}}$, where *x* is distance, t is time and P is power.
12.	If $v =$ velocity of a body and $c =$ speed of light. Write the dimensional formula of $\frac{v}{c}$.
13.	In the relation $\alpha = \beta t + \lambda$., α and λ are measured in metre (m) and t is measured is second (s). What is the S I Unit of β?
	KINEMATICS
14.	Draw a $v - t$ graph to represent a uniform motion. **15.** If a body travels half the distance with velocity v_1 and next half with velocity v_2 then which one of the following will be the average velocity of the body?
16.	A body starting from the rest moves along a straight line with constant acceleration. Draw a velocity – time graph to represent the motion. What does the $v -$ intercept and slope of the graph represent?
17.	A car accelerates from rest with acceleration 1.2 m/s^2 as soon as the bus passes it. The bus moves with constant speed of 12 m/s in a parallel lane. How long does the car take from its start to meet the bus?
18.	A bus moving at a speed of 24 m/s begins to slow at the rate of 3 m/s each second. How far does it go before stopping?
19.	A stone is thrown vertically upwards with an initial velocity u from the top of a tower of height $12u^2/g$. With what velocity does the stone reach the ground?
20.	A particle starts from rest, accelerates uniformly tor 3 seconds and then decelerates uniformly for 3 seconds and comes to rest. Draw a displacement (x) - time (t) graph to represent the motion of the particle.
21.	A motorcycle, initially at rest, is given a constant acceleration for some time and then constant retardation b, till it comes to rest. If the total time elapsed is t seconds, what is the maximum velocity acquired by the motorcycle?

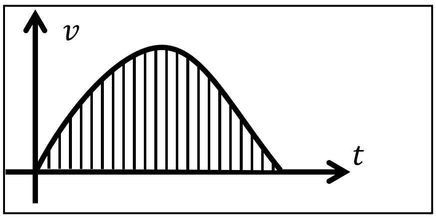
22. The graph (velocity-time) shown represents motion of a particle.



Draw acceleration-time graph for this motion.

23. Let a particle A move from s = 0 at t = 0 along a straight line with an initial velocity v_1, and with a steady acceleration a_1 .Let a particle B move from s = 0 at t = 0 along the same straight line with an initial velocity v_2 and a steady acceleration a_2. If $v_1 < v_2$, and $a_1 > a_2$, then plot a $v - t$ graphs for these motions.

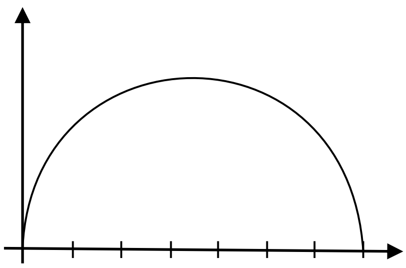
24.



Which characteristic of the particle does the shaded area of the velocity-time graph shown above represent ?

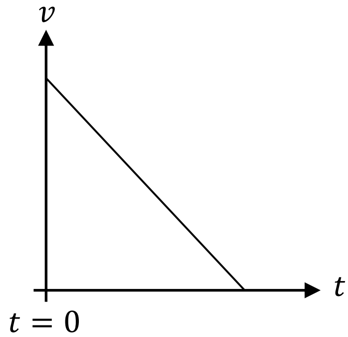
25. In a vacuum, a five-rupee coin, a feather of a sparrow bird and a mango are dropped simultaneously from the same height. The time taken by them to reach the bottom is t_1, t_2 and t_3 respectively. Find the relation among the time of fall.

26. The plot given below represents the velocity of a particle in (m/s) with time (in second).



Assuming that the plot represents a semi-circle, find the distance traversed by the particle at the end of 7 seconds is approximately.

27. What type of motion is represented by the velocity – time plot shown below?



BIOLOGY CLASS - XI

SUMMER VACATION HOME ASSIGNMENT 2025 – 2026

CHAPTER 1

1. Explain about taxonomical aids or tool.
2. From the identification of individuals and populations, what do we learn?
3. Give a reason- 'Why growth and reproduction cannot be taken as defining properties of living beings?
4. Write a short note on zoological parks
5. What are Endemic species and Exotic species? Give examples also.
6. Write down the steps to set up a herbarium? Collect 3 plant species and prepare a herbarium of it.
7. *Brassica compestris Linn*
   i) Give the common name of the plant.
   ii) What do the first two parts of the name denote?
   iii) Why are they written in italics?
   iv) What is the meaning of Linn written at the end of the name?
8. For identification, mention some of the taxonomic aids. Also, discuss in detail taxonomic aids
9. Growth and reproduction are not taken as defining properties of all living beings.
10. What are the three codes of Nomenclature?

CHAPTER 2

1. Discuss how the classification system has undergone several changes over a period of time.
2. Give a detailed account of the classes of Kingdom Fungi under the following:
   (i) Mode of nutrition
   (ii) Mode of reproduction
3. Discuss the salient features of viruses with the help of diagram?
4. Diatoms are also called as ‘pearls of ocean’, why? What is diatomaceous earth?
5. Polluted water bodies contain plants like Nostoc and Oscillitoria. Give reasons.
6. Find out what the terms ‘algal bloom’ and ‘red tides’ signify.
7. Explain the phylogenetic system of classification.
8. Find out what the terms ‘algal bloom’ and ‘red tides’ signify.
9. Explain sexual reproduction in bacteria.

CHAPTER 3

1. Both gymnosperms and angiosperms bear seeds but then why are they classified separately?
2. “Algae and Bryophytes are different from each other.” Point out the main differences between them?
3. Distinguish the reproductive organ of gymnosperm and angiosperm.
4. Write a note on the economic importance of algae and gymnosperm.
5. How are the male gametophytes and female gametophytes of pteridophytes and gymnosperms different?
6. Why bryophytes are also called the amphibians of the plant kingdom?
7. Discuss the phylogenetic relationship of Cycas with any other group of plants.
8. State the different types of patterns seen in the alternation of generation in plants?